Supplementary Technical Document

To prevent numerical cohesion between phases common to MPM, we adopt three separate background MPM grids, one for volumetric elastic and rigid objects, one for general MPM materials, and one for them combined. We denote quantities associated with traditional MPM particles with subscript MPM, p, quantities associated with collision particles with subscript q, quantities associated with combined grid with subscript $\mathbf{i}$, quantities associated with volumetric grid $vol, \mathbf{i}$, and quantities associated with MPM grid $MPM, \mathbf{i}$. So we have

$$m_{vol,\mathbf{i}}^n = \sum_q w_{\mathbf{i}q}^n m_q \tag{1}$$

$$m_{MPM,\mathbf{i}}^n = \sum_{MPM,p} w_{\mathbf{i}p}^n m_{MPM,p} \tag{2}$$

$$m_{\mathbf{i}}^n = m_{vol,\mathbf{i}}^n + m_{MPM,\mathbf{i}}^n \tag{3}$$

$$\mathbf{v}_{vol,\mathbf{i}}^* = \frac{1}{m_{vol,\mathbf{i}}^n} \sum_q w_{\mathbf{i}q}^n m_q \mathbf{v}_q^* \tag{4}$$

$$\mathbf{v}_{MPM,\mathbf{i}}^n = \frac{\sum_{MPM,p} w_{\mathbf{i}p}^n m_{MPM,p} \left(\mathbf{v}_{MPM,p} + \mathbf{C}_{MPM,p}(\mathbf{x}_{\mathbf{i}} - \mathbf{x}_{MPM,p})\right)}{m_{MPM,\mathbf{i}}^n} \tag{5}$$

$$\mathbf{v}_{\mathbf{i}}^n = \frac{m_{vol,\mathbf{i}}^n \mathbf{v}_{vol,\mathbf{i}}^* + m_{MPM,\mathbf{i}}^n \mathbf{v}_{MPM,\mathbf{i}}^n}{m_{\mathbf{i}}^n} \tag{6}$$

$$\mathbf{n}_{\mathbf{i}}^n = \frac{\sum_q w_{\mathbf{i}q} \mathbf{n}_q}{\| \sum_q w_{\mathbf{i}q} \mathbf{n}_q \|} \tag{7}$$

Grid velocity $\mathbf{v}_{MPM,\mathbf{i}}$ is updated as in [1, 2] to get $\mathbf{v}_{MPM,\mathbf{i}}^*$. Then the collision between phases is handled through an inelastic collision on collocated grid nodes. Let $\mathbf{v}_r = \mathbf{v}_{vol,\mathbf{i}}^* - \mathbf{v}_{MPM,\mathbf{i}}^n$. If $\mathbf{v}_r \cdot \mathbf{n}_{\mathbf{i}} < 0$, we apply impulse in the following way:

$$\mathbf{v}_t = \mathbf{v}_r - \mathbf{v}_r \cdot \mathbf{n}_{\mathbf{i}}^n \mathbf{n}_{\mathbf{i}}^n \tag{8}$$

$$I_{\mathbf{i}} = \frac{m_{MPM,\mathbf{i}}^n m_{vol,\mathbf{i}}^n}{m_{MPM,\mathbf{i}}^n + m_{vol,\mathbf{i}}^n} \mathbf{v}_r \cdot \mathbf{n}_q^n \tag{9}$$

$$\mathbf{v}_{MPM,\mathbf{i}}^{n+1} = \mathbf{v}_{MPM,\mathbf{i}}^* + \frac{I_{\mathbf{i}} \mathbf{n}_{\mathbf{i}}}{m_{MPM,\mathbf{i}}^n} + \min\left(-\frac{\mu I_{\mathbf{i}}}{m_{MPM,\mathbf{i}}^n}, \|\mathbf{v}_t\|\right) \frac{\mathbf{v}_t}{\|\mathbf{v}_t\|} \tag{10}$$

Finally, we interpolate the the grid velocity $\mathbf{v}_{MPM,\mathbf{i}}^{n+1}$ to MPM particles with APIC as in [1, 2], and the updated velocity of collision particle q is then

$$\mathbf{v}_q^\star = \sum_{\mathbf{i}} w_{\mathbf{i}q}^n \mathbf{v}_{\mathbf{i}}^n. \tag{11}$$

References

[1] C. Jiang, C. Schroeder, A. Selle, J. Teran, and A. Stomakhin. The affine particle-in-cell method. *ACM Trans Graph*, 34(4):51:1–51:10, 2015.

[2] Chenfanfu Jiang, Craig Schroeder, Joseph Teran, Alexey Stomakhin, and Andrew Selle. The material point method for simulating continuum materials. In *ACM SIGGRAPH 2016 Course*, pages 24:1–24:52, 2016.