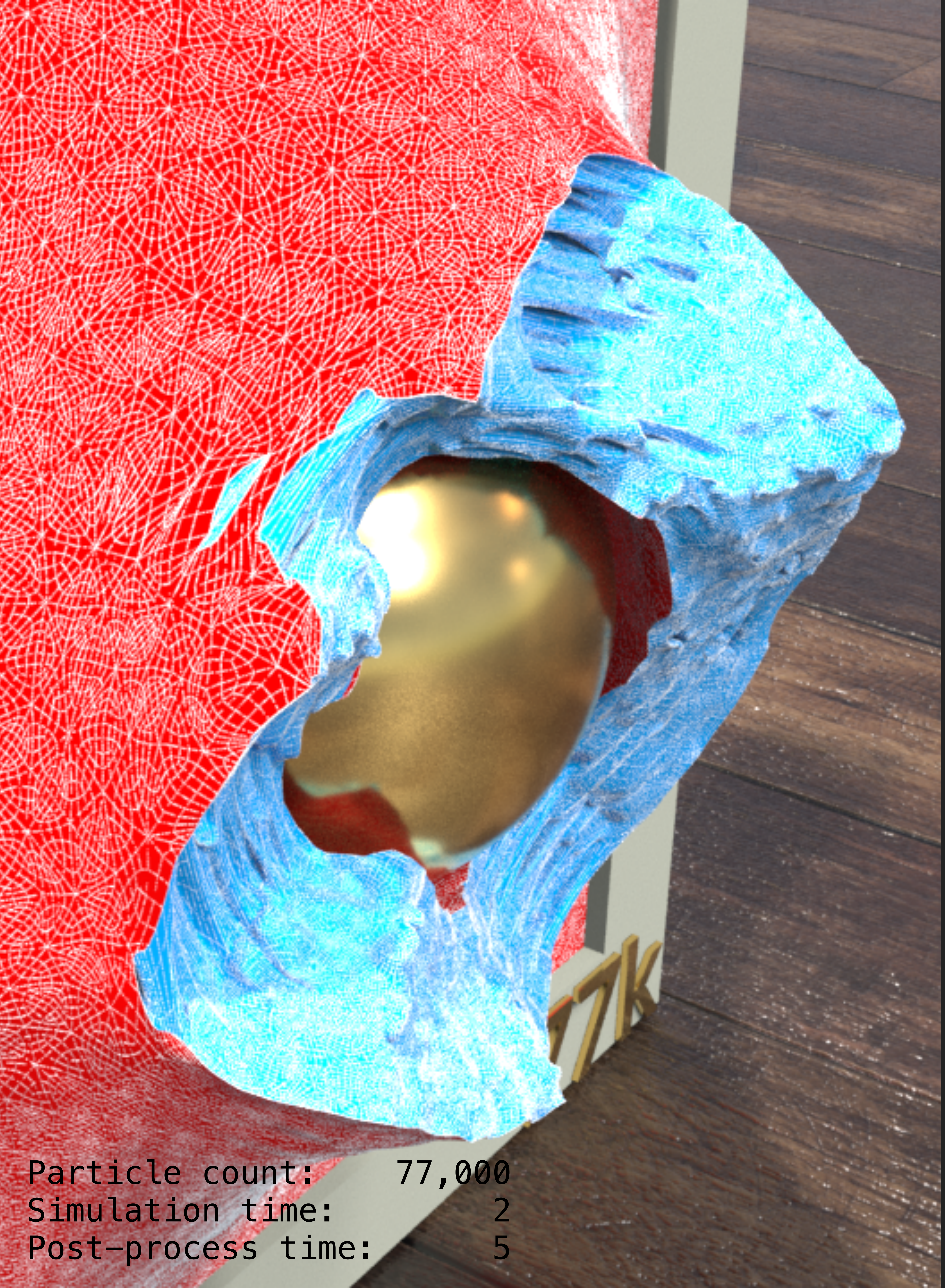

Stephanie Wang
University of California — Los Angeles
August 28, 2019

SIMULATION AND VISUALIZATION OF DUCTILE FRACTURE WITH MATERIAL POINT METHOD (MPM)

ADVISOR & COLLABORATORS

- Advisor: Joseph Teran, UCLA
- Mengyuan Ding, UCLA
- Theodore Gast, JIXIE EFFECTS (UCLA)
- Leyi Zhu, University of Science and Technology of China
- Steven Gagniere, UCLA
- Chenfanfu Jiang, University of Pennsylvania (UCLA)


Particle count: 60,000
Simulation time: 11
Post-process time: N/A


Particle count: 60,000
Simulation time: 11
Post-process time: 5


Particle count: 207,000
Simulation time: 16
Post-process time: 13

OUTLINE

- Material Point Method (MPM)
  - Grid-particle transfer
  - Force computation
- Simulation and visualization of ductile fracture
  - Yield surfaces
  - Mesh-processing
- Discussion

THE MATERIAL POINT METHOD

ROUGH ALGORITHM

- Particles for state
- Grid for computations
- Similar to FEM:
  - Vertices for state
  - Mesh for computations
- Interpolation between particles and grid

ROUGH ALGORITHM

$$m_i^n = \textsc{TransferP2G}(m_p)$$
$$\mathbf{v}_i^n = \textsc{TransferP2G}(\mathbf{v}_p^n)$$
$$\mathbf{f}_i^n = \textsc{ComputeForce}()$$
$$\tilde{\mathbf{v}}_i^{n+1} = \mathbf{v}_i^n + \frac{\Delta t}{m_i^n}\mathbf{f}_i^n$$
$$\mathbf{v}_p^{n+1} = \textsc{TransferG2P}(\tilde{\mathbf{v}}_i^{n+1})$$
$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \mathbf{v}_p^{n+1}$$

notation	meaning	when	where
$\mathbf{x}_p^{n+1}$	position	after forces	particle
$\mathbf{v}_i^n$	velocity	before forces	grid
m_p	mass	never changes	particle

ROUGH ALGORITHM

$$m_i^n = \textsc{TransferP2G}(m_p)$$
$$\mathbf{v}_i^n = \textsc{TransferP2G}(\mathbf{v}_p^n)$$
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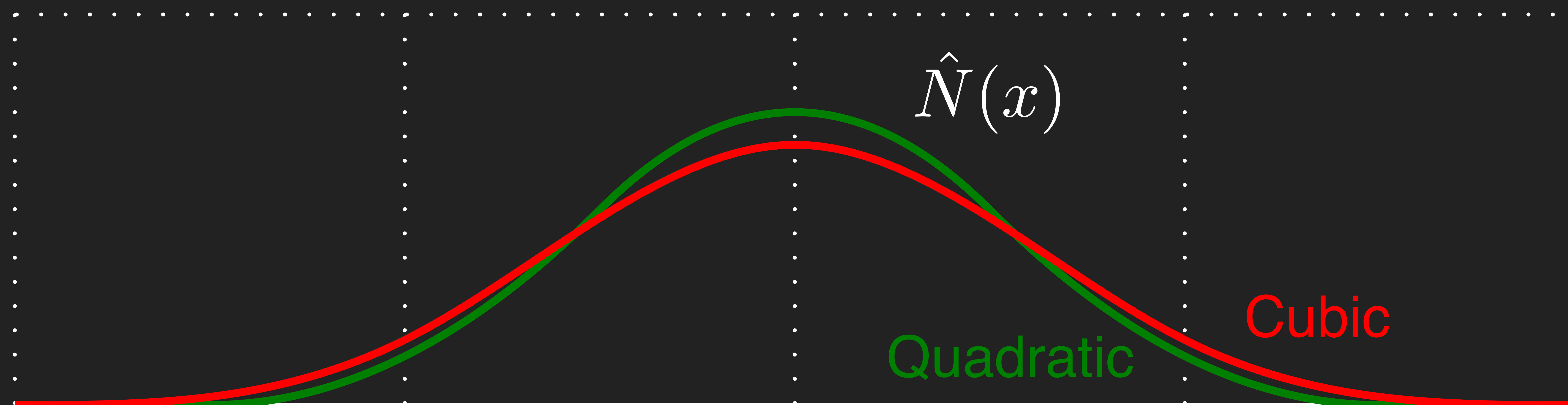
$$\tilde{\mathbf{v}}_i^{n+1} = \mathbf{v}_i^n + \frac{\Delta t}{m_i^n}\mathbf{f}_i^n$$
$$\mathbf{v}_p^{n+1} = \textsc{TransferG2P}(\tilde{\mathbf{v}}_i^{n+1})$$
$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \mathbf{v}_p^{n+1}$$

notation	meaning	when	where
$\mathbf{x}_p^{n+1}$	position	after forces	particle
$\mathbf{v}_i^n$	velocity	before forces	grid
m_p	mass	never changes	particle

INTERPOLATION SCHEME

- Compactly supported kernel function
- Spline: C1 (C2) piecewise-polynomial



INTERPOLATION SCHEME

- Tensor product: $N(\mathbf{x}) = \hat{N}(x)\hat{N}(y)\hat{N}(z)$
- Compute weights: $w_{ip}^n = N(\mathbf{x}_i^n - \mathbf{x}_p^n)$

  $\nabla w_{ip}^n = \nabla N(\mathbf{x}_i^n - \mathbf{x}_p^n)$
- Partition of unity $\sum_i w_{ip}^n = 1$
- Barycentric embedding $\sum_i w_{ip}^n \mathbf{x}_i^n = \mathbf{x}_p^n$
- Conservation of momenta, non-increasing energy

INTERPOLATION SCHEME

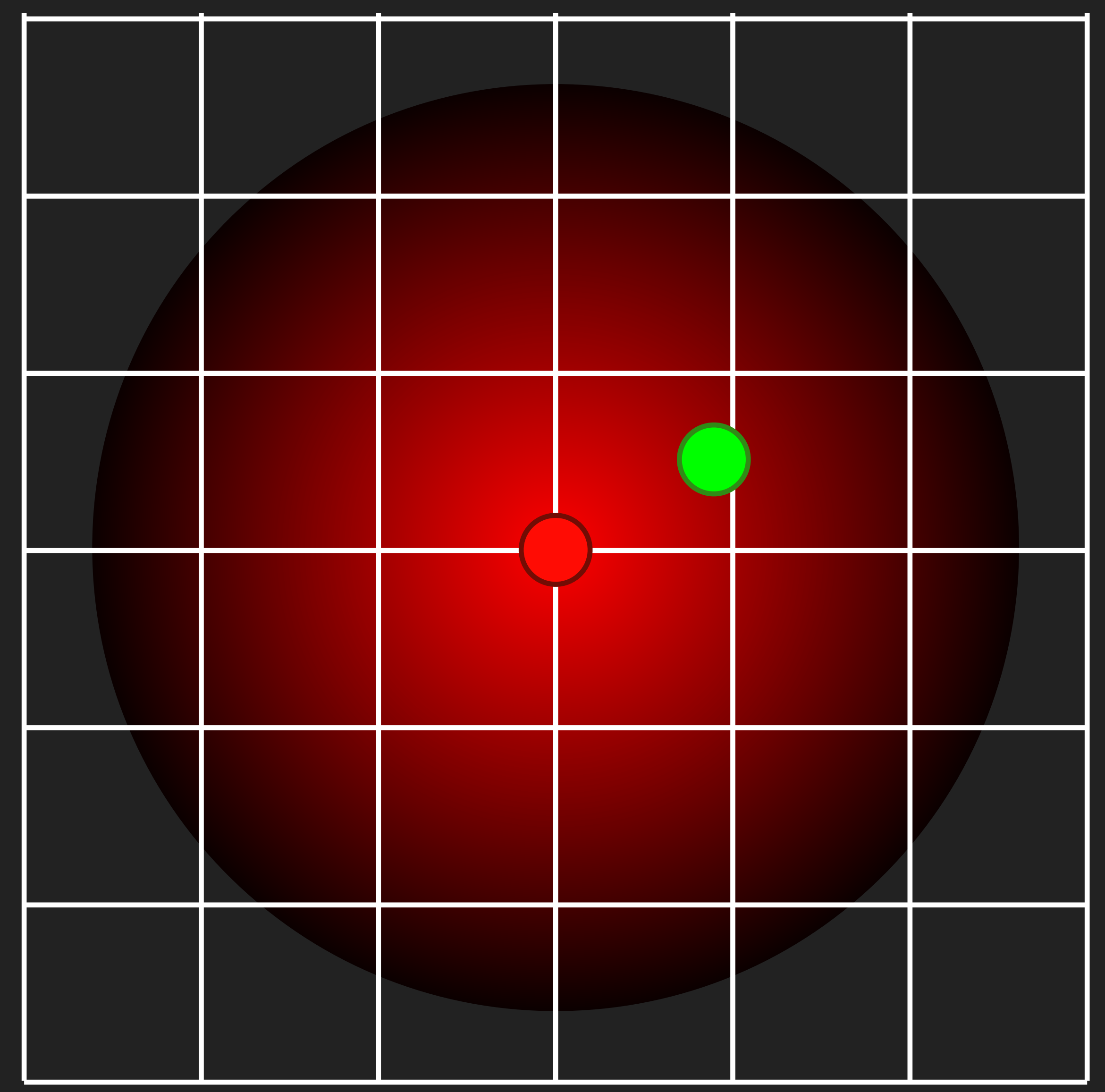
TransferP2G

$$m_i^n = \sum_p w_{ip}^n m_p$$ Mass

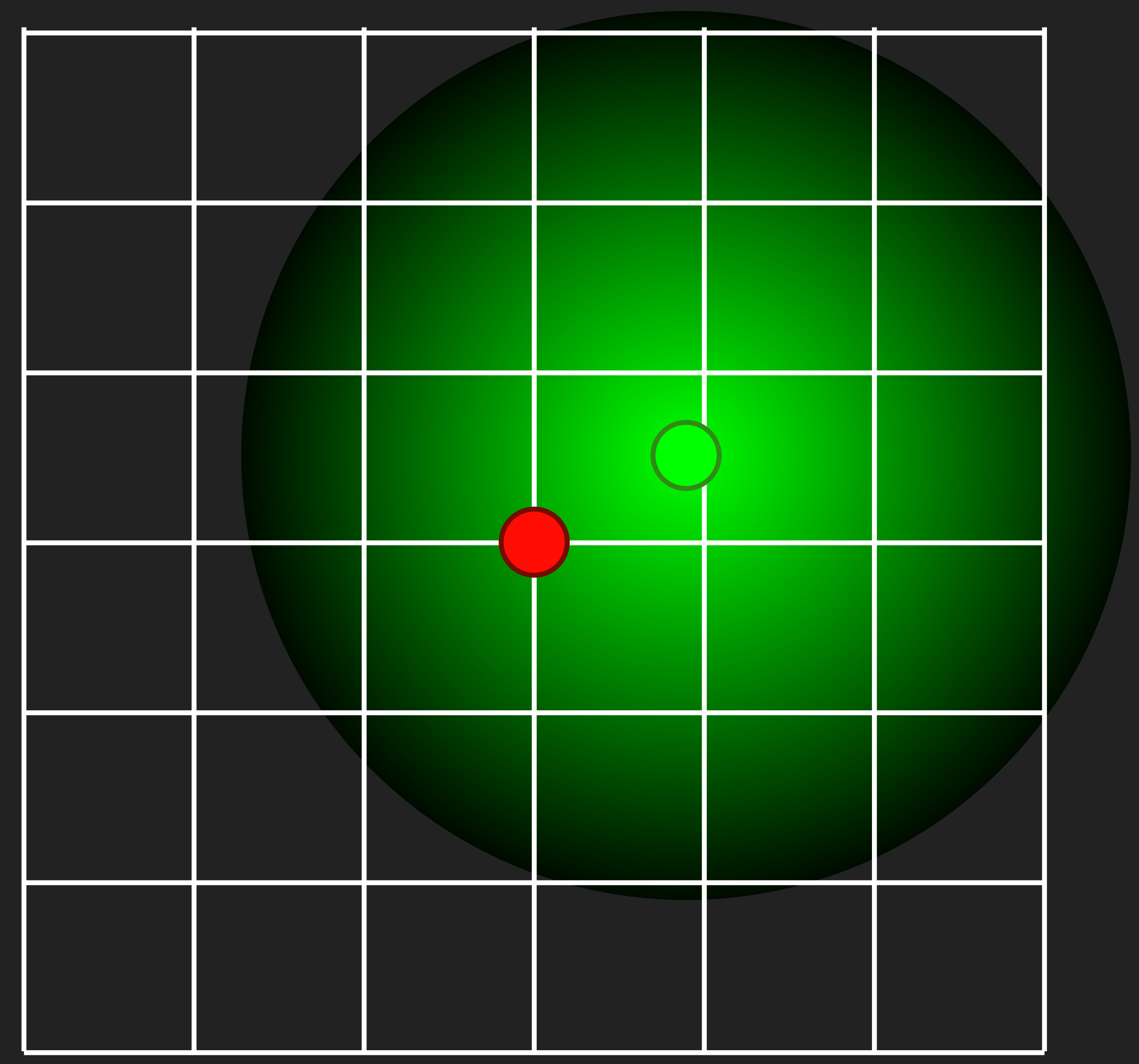
$$m_i^n \mathbf{v}_i^n = \sum_p w_{ip}^n m_p \mathbf{v}_p^n$$ Momentum

TransferG2P

$$\mathbf{v}_p^{n+1} = \sum_i w_{ip}^n \tilde{\mathbf{v}}_i^{n+1}$$



Kernel at node



Kernel at particle

PIC, FLIP, APIC, RPIC, ...

$$m_i^n = \sum_p w_{ip}^n m_p$$

$$\mathbf{v}_i^n = \frac{1}{m_i^n} \sum_p w_{ip}^n m_p \mathbf{v}_p^n$$

$$\mathbf{f}_i^n = \textsc{ComputeForce}()$$

$$\tilde{\mathbf{v}}_i^{n+1} = \mathbf{v}_i^n + \frac{\Delta t}{m_i^n} \mathbf{f}_i^n$$

$$\mathbf{v}_p^{n+1} = \sum_i w_{ip}^n \tilde{\mathbf{v}}_i^{n+1}$$

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \mathbf{v}_p^{n+1}$$

Particle In Cell (PIC)

$$m_i^n = \sum_p w_{ip}^n m_p$$

$$\mathbf{D}_p^n = \sum_i w_{ip}^n (\mathbf{x}_i^n - \mathbf{x}_p^n)(\mathbf{x}_i^n - \mathbf{x}_p^n)^T$$

$$\mathbf{v}_i^n = \frac{1}{m_i^n} \sum_p w_{ip}^n m_p (\mathbf{v}_p^n + \mathbf{B}_p^n (\mathbf{D}_p^n)^{-1} (\mathbf{x}_i^n - \mathbf{x}_p^n))$$

$$\mathbf{f}_i^n = \textsc{ComputeForce}()$$

$$\tilde{\mathbf{v}}_i^{n+1} = \mathbf{v}_i^n + \frac{\Delta t}{m_i^n} \mathbf{f}_i^n$$

$$\mathbf{v}_p^{n+1} = \sum_i w_{ip}^n \tilde{\mathbf{v}}_i^{n+1}$$

$$\mathbf{B}_p^{n+1} = \sum_i w_{ip}^n \mathbf{v}_i^n (\mathbf{x}_i^n - \mathbf{x}_p^n)^T$$

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \mathbf{v}_p^n$$

Affine Particle In Cell (APIC)

PIC, FLIP, APIC, RPIC, …….

- Particle In Cell (PIC): Harlow 1964
- Fluid Implicit Particle (FLIP): Brackbill and Ruppel 1986
- Affine Particle In Cell (APIC): Jiang et al. 2015
- Rigid Particle In Cell (RPIC): Jiang et al. 2015
- Polynomial Particle In Cell (PolyPIC): Fu et al. 2017
- Extended Particle In Cell (XPIC): Hammerquist et al. 2017

ROUGH ALGORITHM

$$
\begin{aligned}
m_i^n &= \textsc{TransferP2G}(m_p) \\
\mathbf{v}_i^n &= \textsc{TransferP2G}(\mathbf{v}_p^n) \\
\mathbf{f}_i^n &= \textsc{ComputeForce}() \\
\tilde{\mathbf{v}}_i^{n+1} &= \mathbf{v}_i^n + \frac{\Delta t}{m_i^n}\mathbf{f}_i^n \\
\mathbf{v}_p^{n+1} &= \textsc{TransferG2P}(\tilde{\mathbf{v}}_i^{n+1}) \\
\mathbf{x}_p^{n+1} &= \mathbf{x}_p^n + \Delta t \mathbf{v}_p^{n+1}
\end{aligned}
$$

notation	meaning	when	where
$\mathbf{x}_p^{n+1}$	velocity	before forces	grid
$\mathbf{v}_i^n$	position	after forces	particle
m_p	mass	never changes	particle

ROUGH ALGORITHM

$$m_i^n = \textsc{TransferP2G}(m_p)$$
$$\mathbf{v}_i^n = \textsc{TransferP2G}(\mathbf{v}_p^n)$$
$$\boxed{\mathbf{f}_i^n = \textsc{ComputeForce}()}$$
$$\tilde{\mathbf{v}}_i^{n+1} = \mathbf{v}_i^n + \frac{\Delta t}{m_i^n}\mathbf{f}_i^n$$
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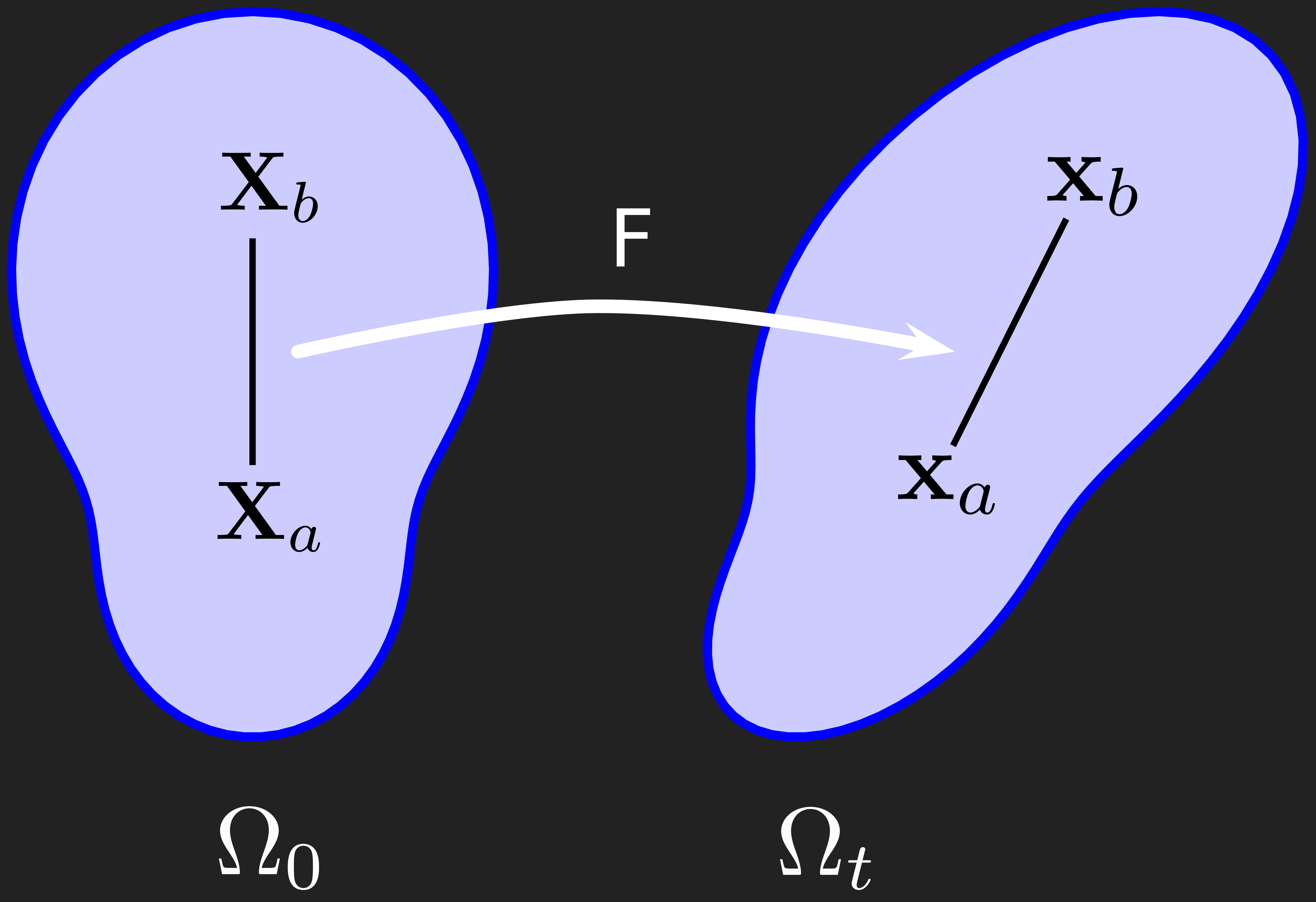
$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \mathbf{v}_p^{n+1}$$

notation	meaning	when	where
$\mathbf{x}_p^{n+1}$	velocity	before forces	grid
$\mathbf{v}_i^n$	position	after forces	particle
m_p	mass	never changes	particle

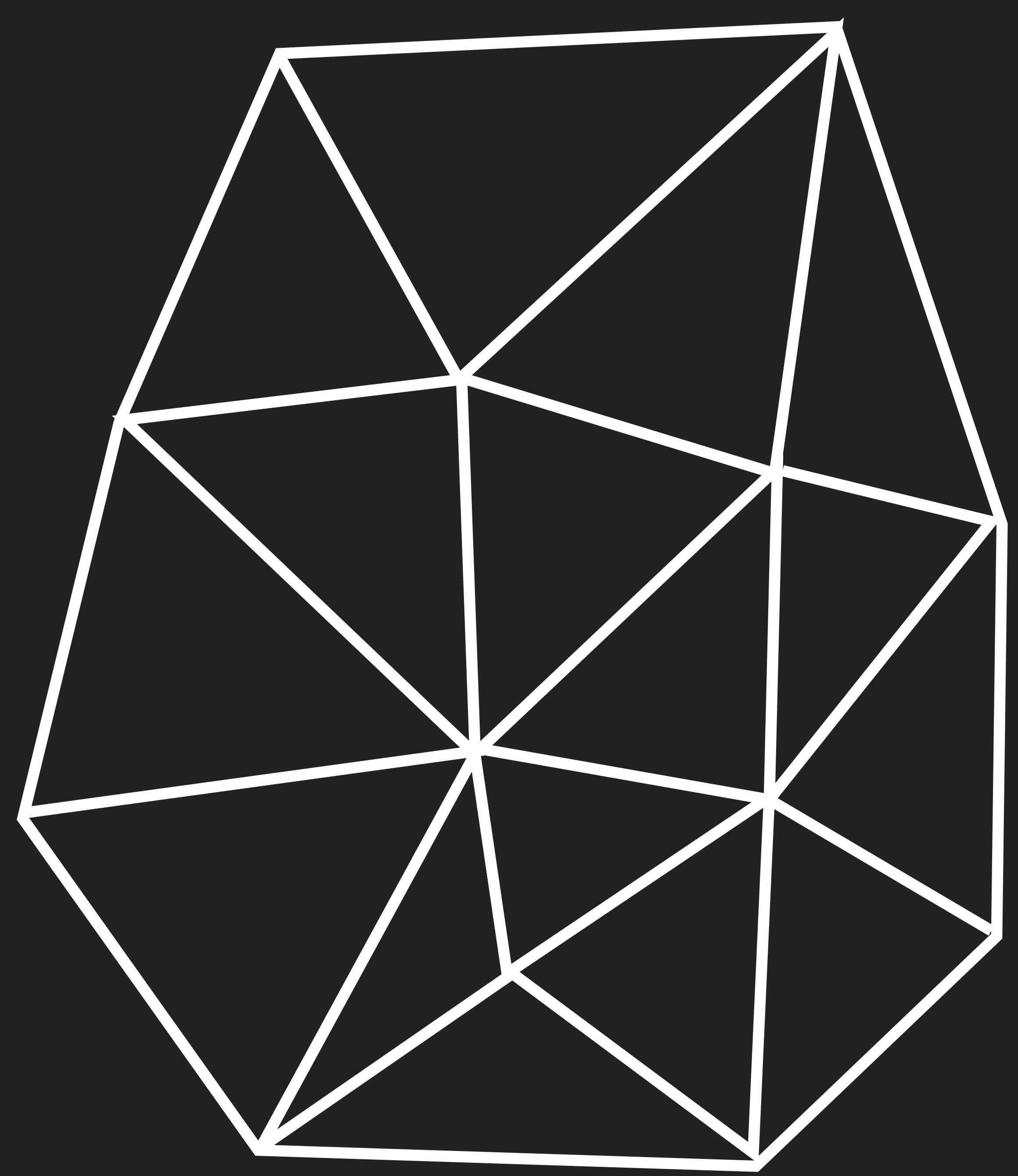
DEFORMATION GRADIENT

$$\mathbf{x} = \Phi(\mathbf{X}, t)$$

$$\mathbf{F}(\mathbf{X}, t) = \frac{\partial \Phi}{\partial \mathbf{X}}(\mathbf{X}, t)$$

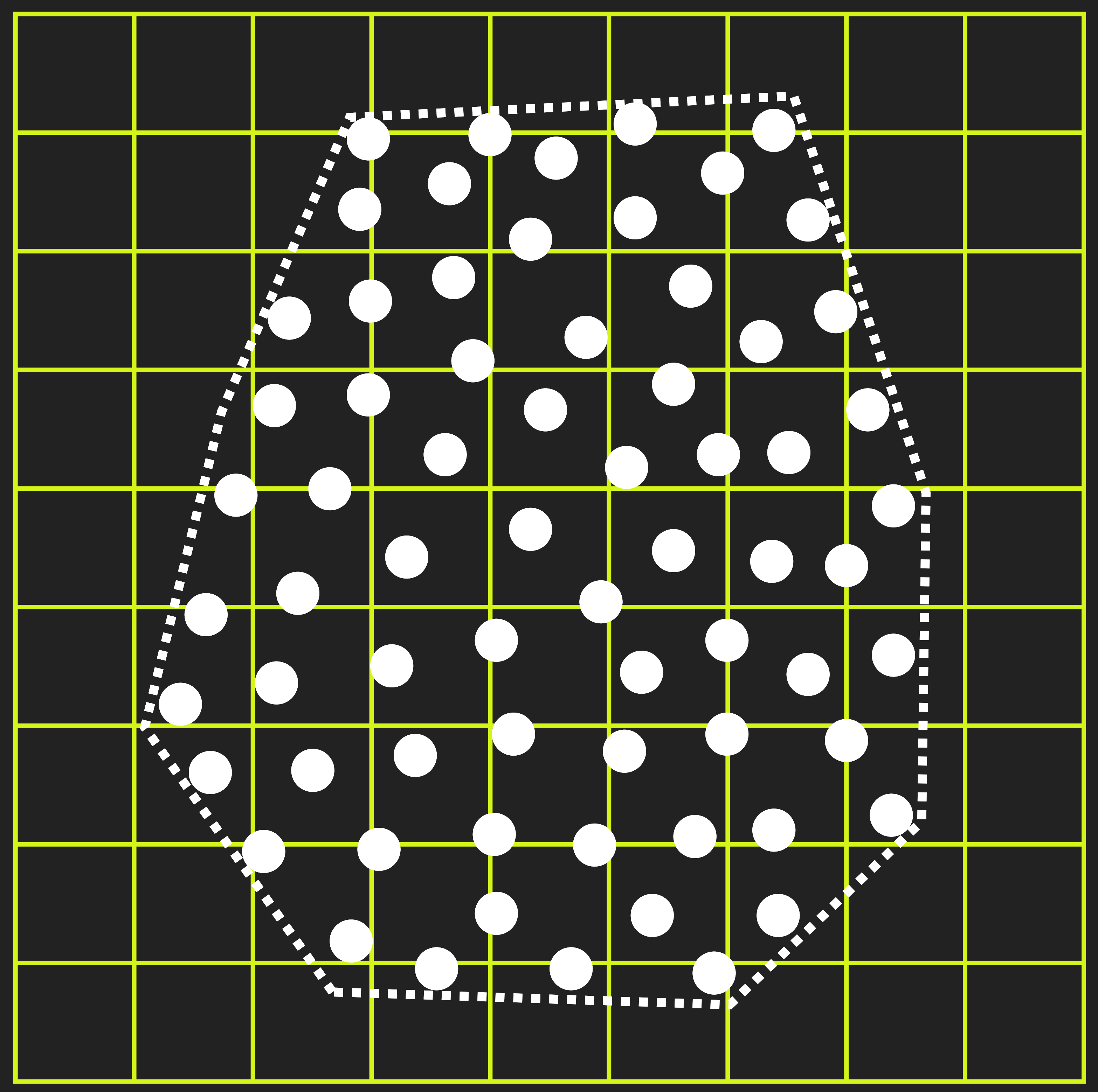


DEFORMATION GRADIENT



mesh-based forces:
F per triangle

$$\Phi = \sum_e V_e^0 \Psi(\mathbf{F}_e)$$



particle-based forces:
F per particle

$$\Phi = \sum_p V_p^0 \Psi(\mathbf{F}_p)$$

FORCE AS ENERGY GRADIENT

- First Piola-Kirchoff stress $\mathbf{P}(\mathbf{F}) = \frac{\partial \Psi}{\partial \mathbf{F}}(\mathbf{F})$
- Total potential energy $\Phi = \sum_p V_p^0 \Psi(\mathbf{F}_p)$
  - "F is a function of x" $\mathbf{F}_p^{n+1} = \left(\mathbf{I} + \Delta t \sum_i \mathbf{v}_i (\nabla \omega_{ip}^n)^T\right) \mathbf{F}_p^n$ $\quad \mathbf{F}_e^n = \sum_q \mathbf{x}_q^n \nabla N_q(\mathbf{X}_e)^T$
  - Energy is a function of x $\mathbf{f}_i = -\frac{\partial \Phi}{\partial \mathbf{x}_i}$
  - Force can be computed from x

$$\mathbf{f}_i = -\frac{\partial \Phi}{\partial \mathbf{x}_i} = -\sum_p V_p^0 \left(\frac{\partial \Psi}{\partial \mathbf{F}}(\mathbf{F}_p(\mathbf{x}))\right) (\mathbf{F}_p^n)^T \nabla \omega_{ip}^n$$

HYPER-ELASTIC MODELS

- St. Venant Kirchhoff potential with Hencky strain

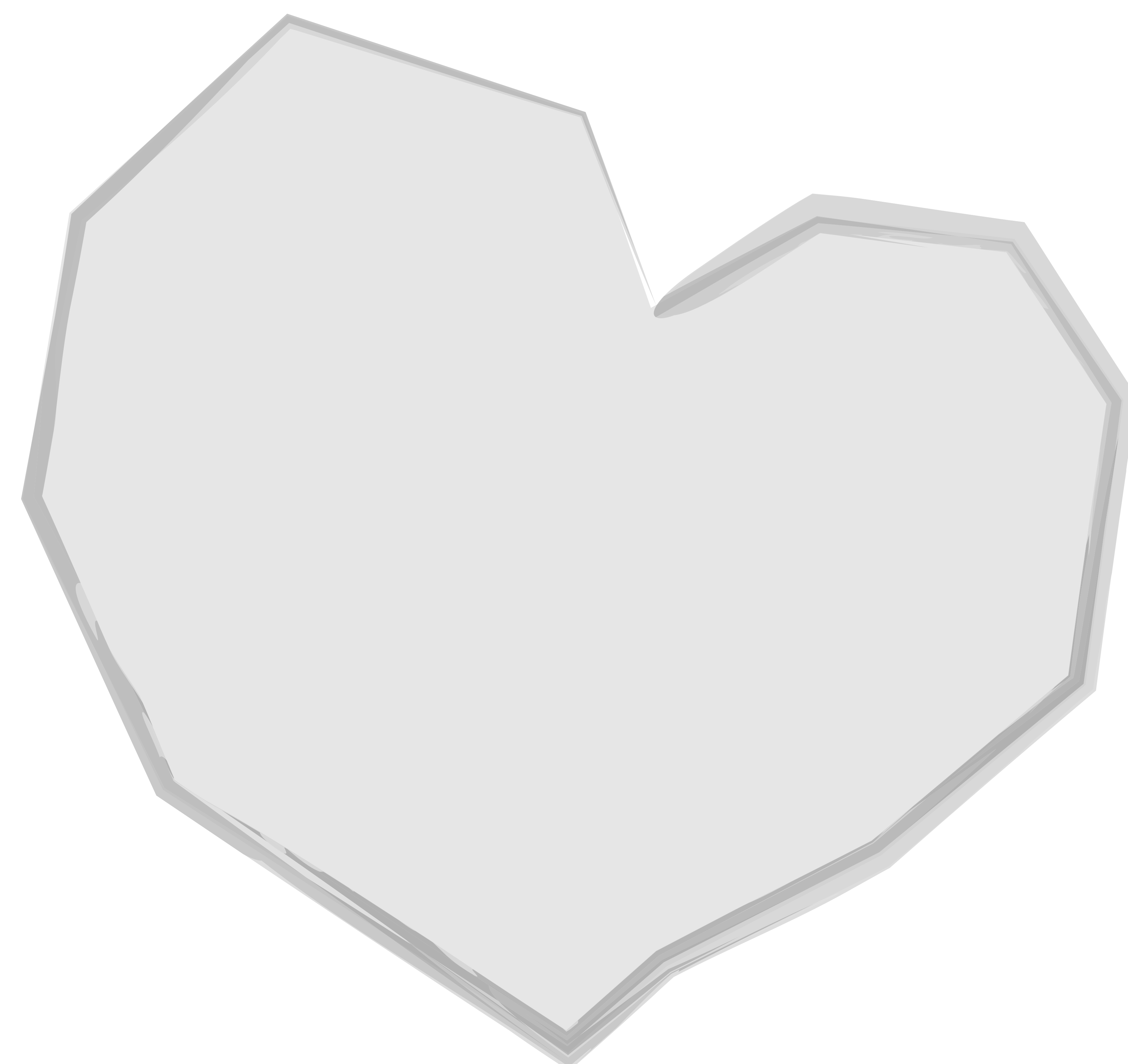
$$\mathbf{F} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$$

$$\psi(\mathbf{F}) = \mu \mathrm{tr}((\ln \boldsymbol{\Sigma})^2) + \frac{\lambda}{2}(\mathrm{tr}(\ln \boldsymbol{\Sigma}))^2$$

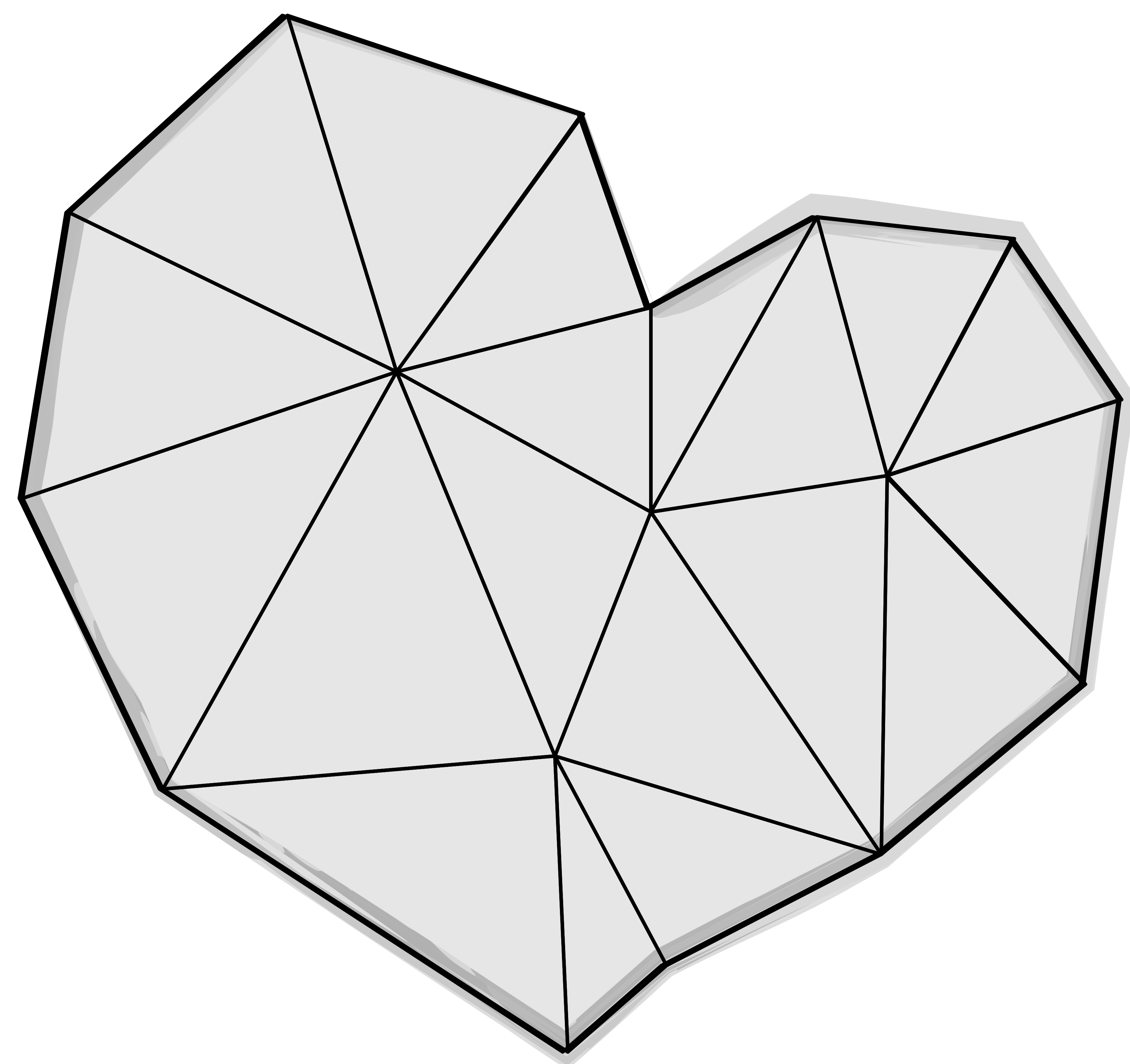
$$\frac{\partial \psi}{\partial \mathbf{F}} = \mathbf{U}(2\mu\boldsymbol{\Sigma}^{-1}\ln \boldsymbol{\Sigma} + \lambda \mathrm{tr}(\ln \boldsymbol{\Sigma})\boldsymbol{\Sigma}^{-1})\mathbf{V}^T$$

- (Easy for analytical plastic projection)

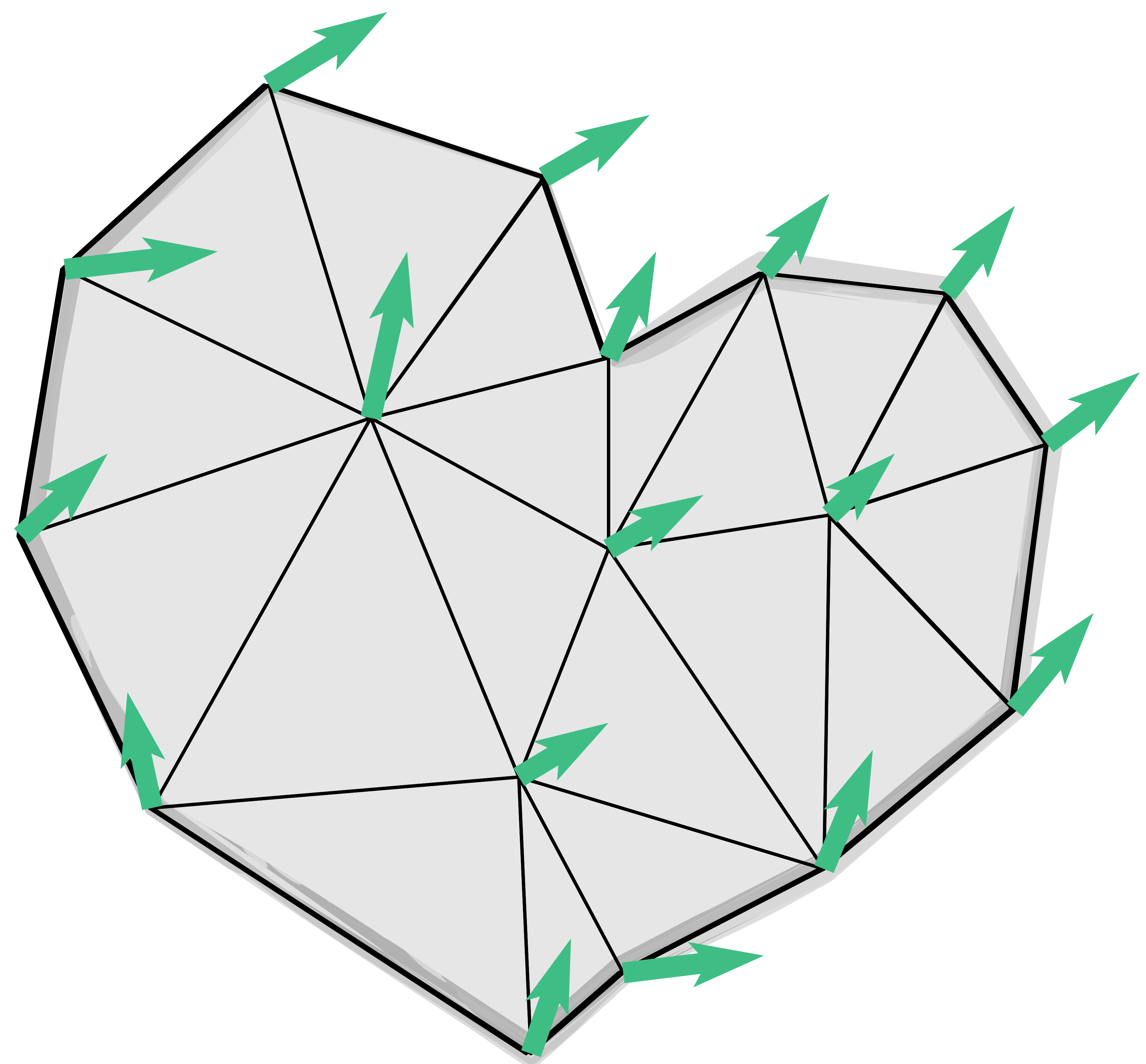
FINITE ELEMENT ELEMENT



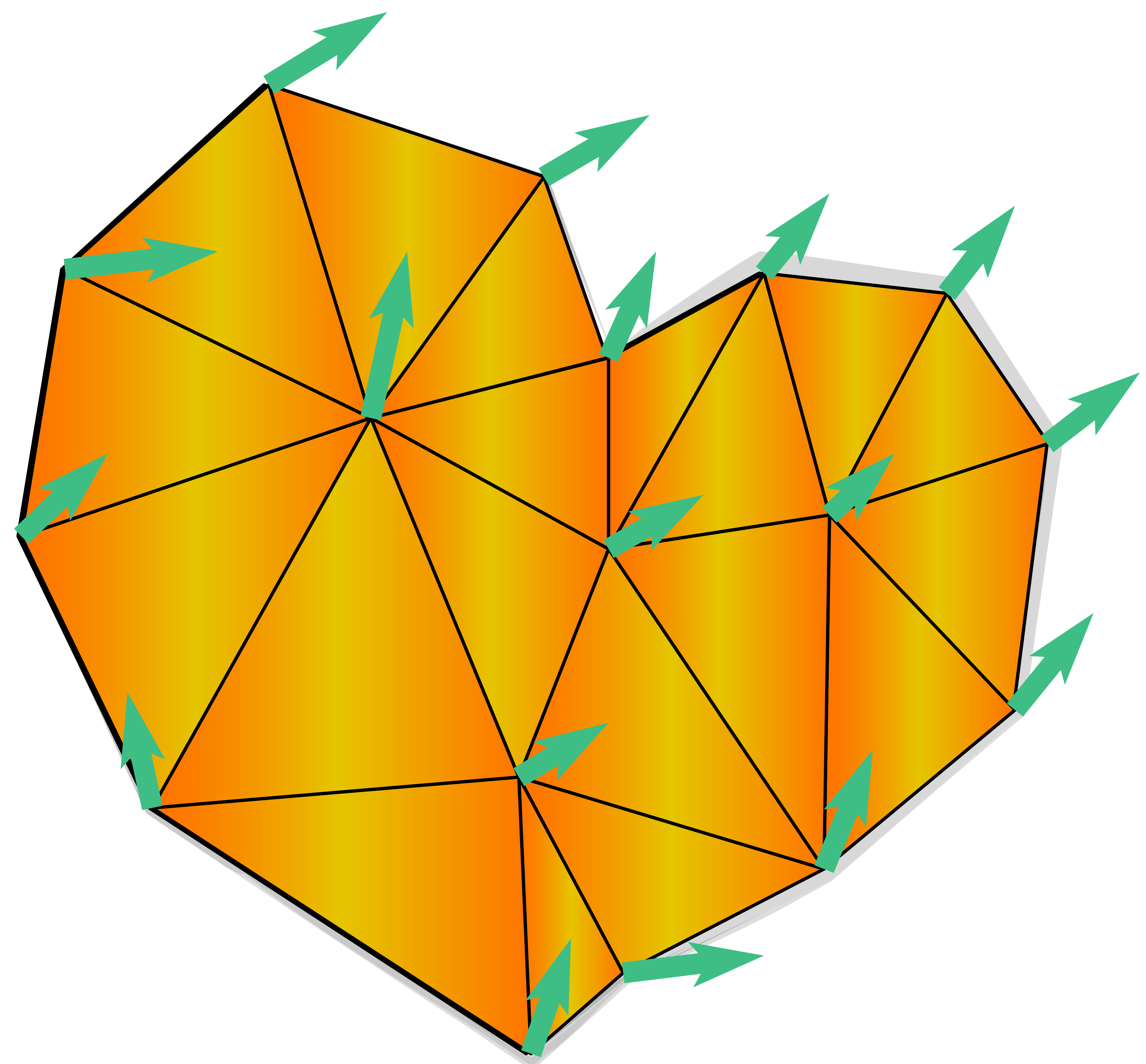
FINITE ELEMENT ELEMENT



FINITE ELEMENT ELEMENT



FINITE ELEMENT ELEMENT

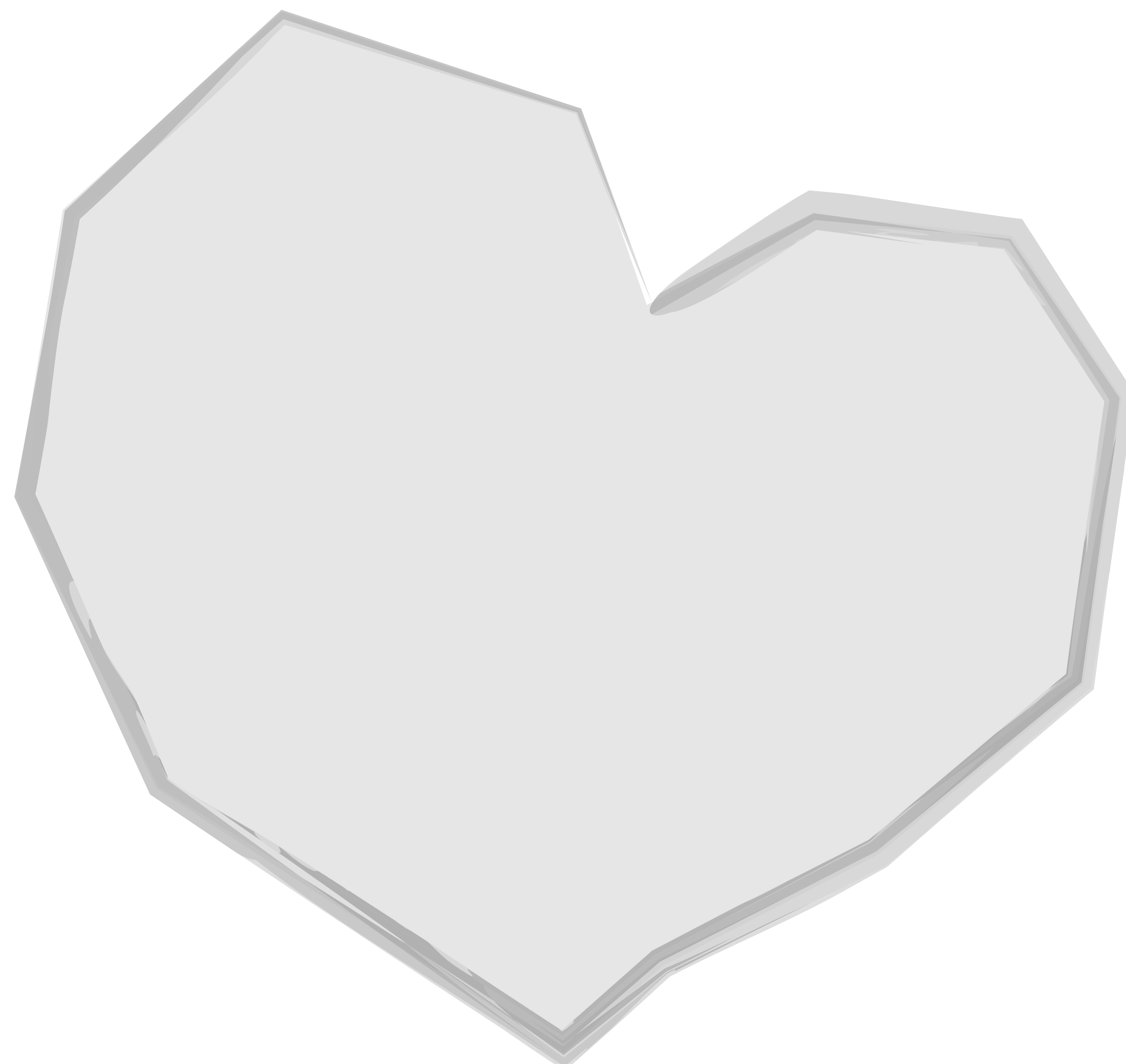


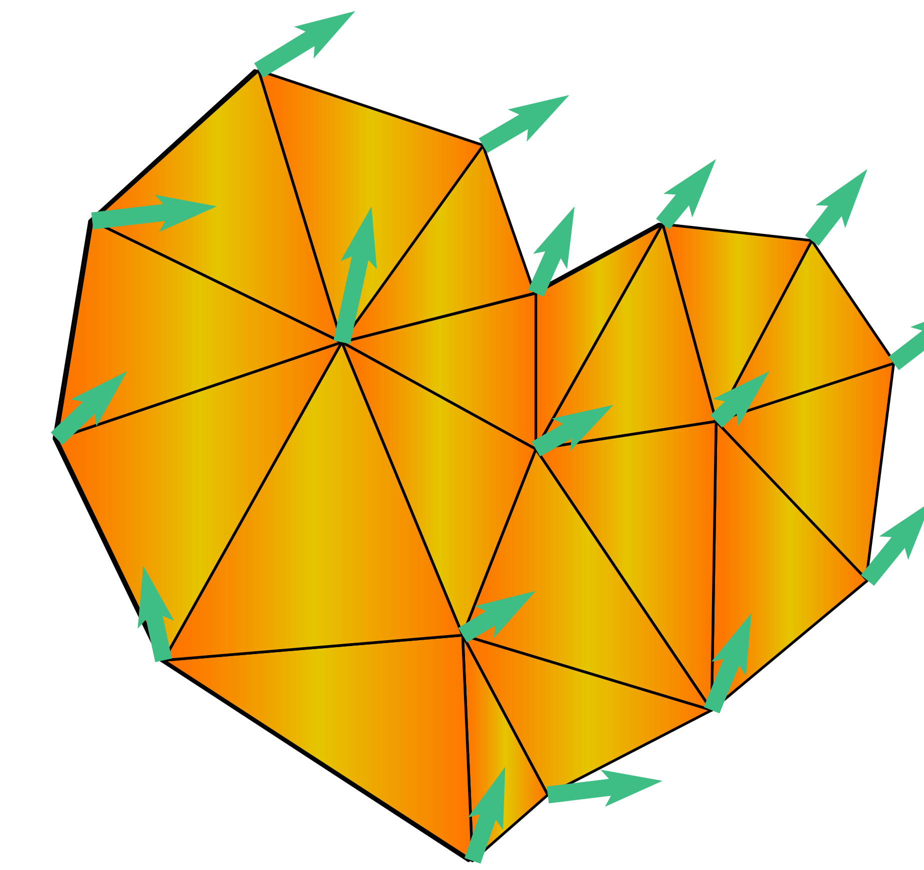
$$\Phi = \sum_e V_e^0 \Psi(\mathbf{F}_e)$$

$$\mathbf{F}_e^n = \sum_q \mathbf{x}_q^n \nabla N_q(\mathbf{X}_e)^T$$

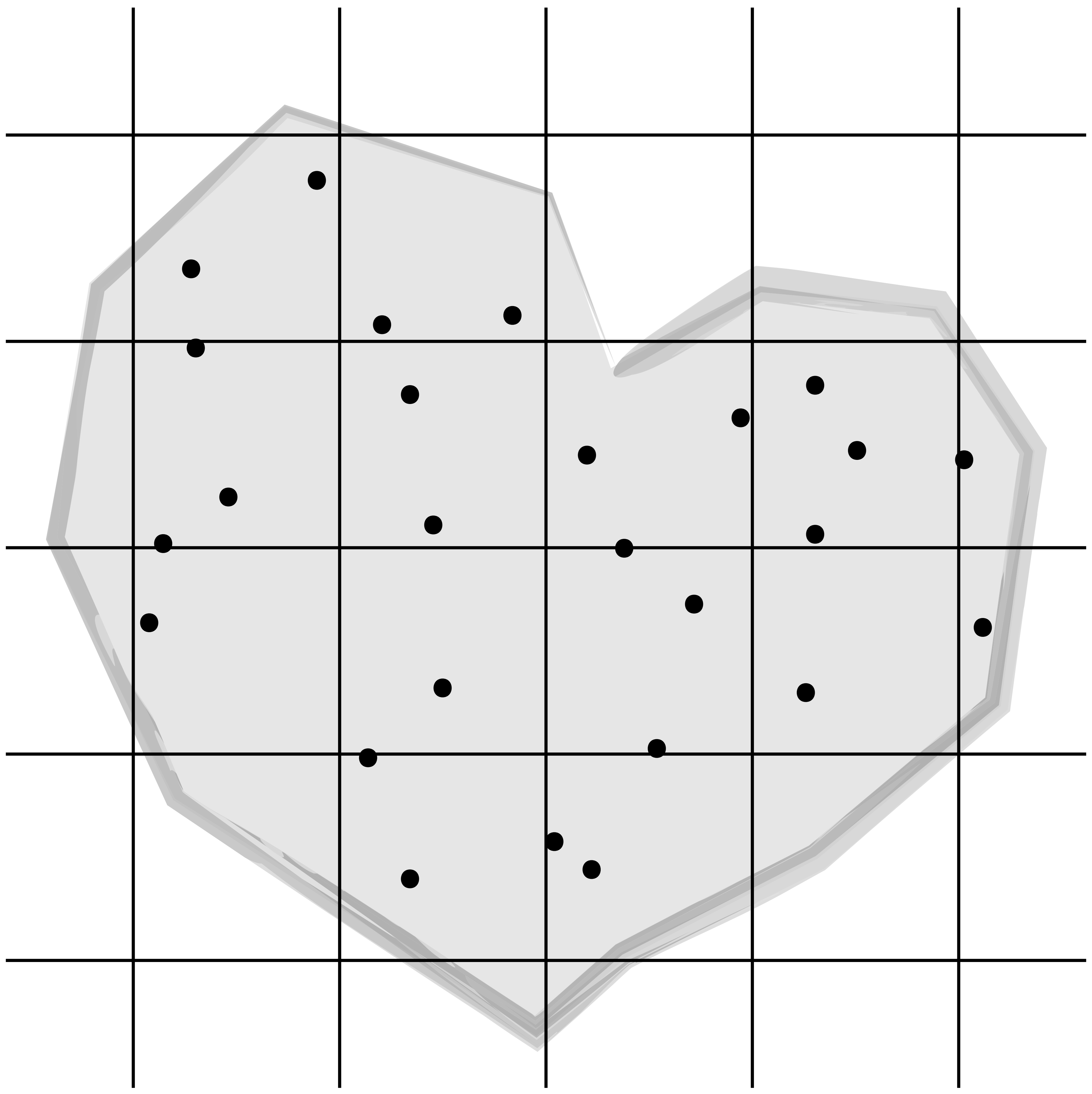
$$\mathbf{F}_e^n = \left(\sum_q \mathbf{x}_q^n \nabla N_q(\xi_e)^T\right)\left(\sum_q \mathbf{X}_q \nabla N_q(\xi_e)^T\right)^{-1}$$

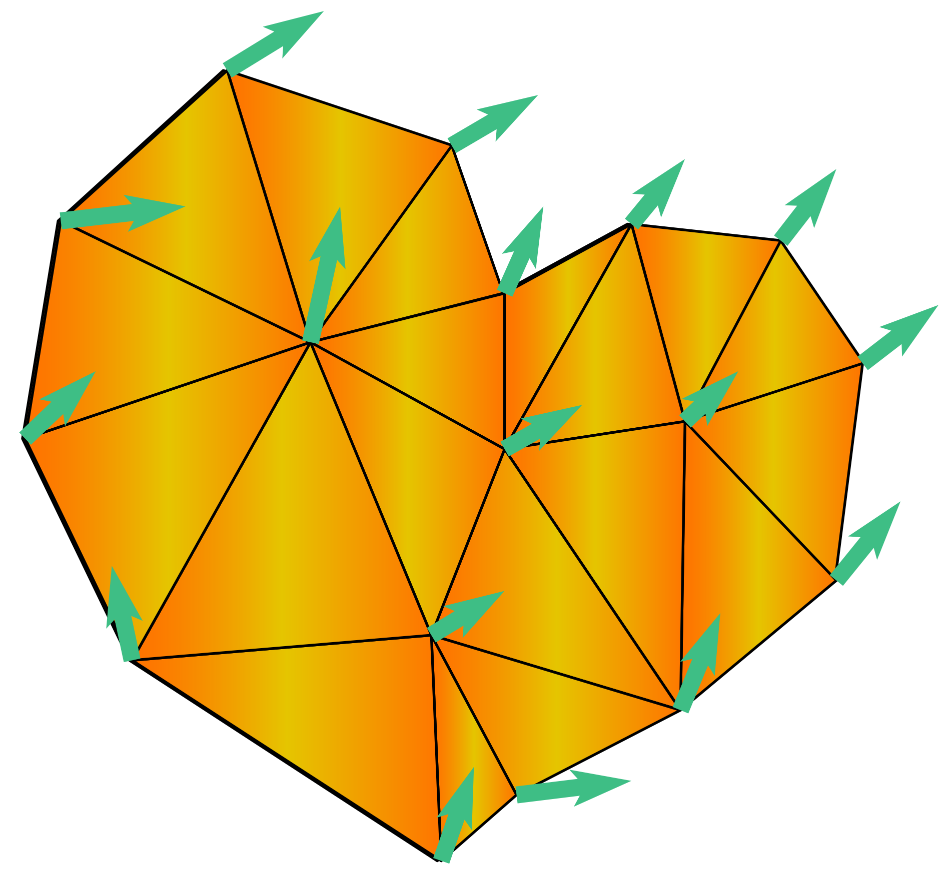
PARTICLE MPM



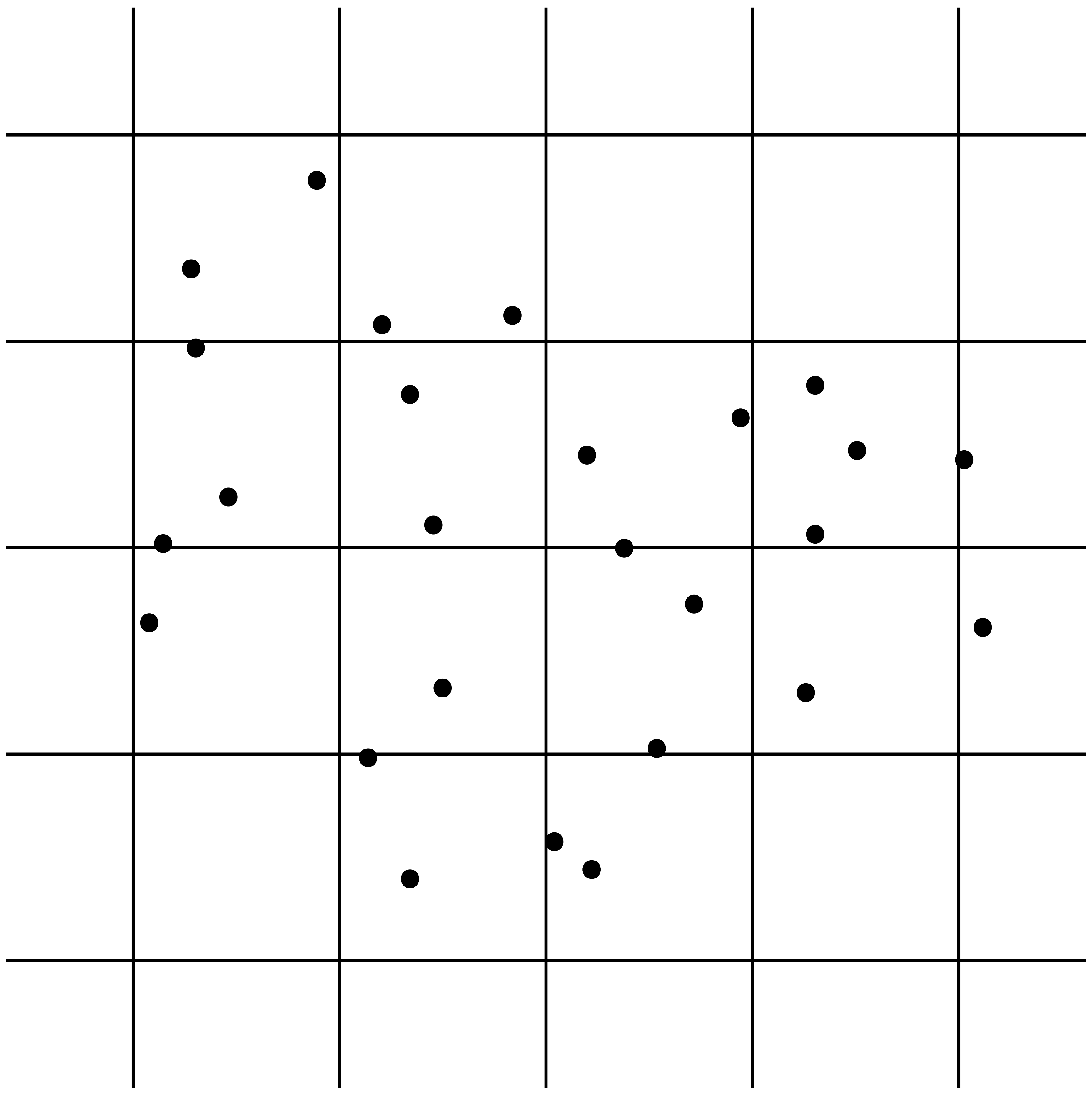


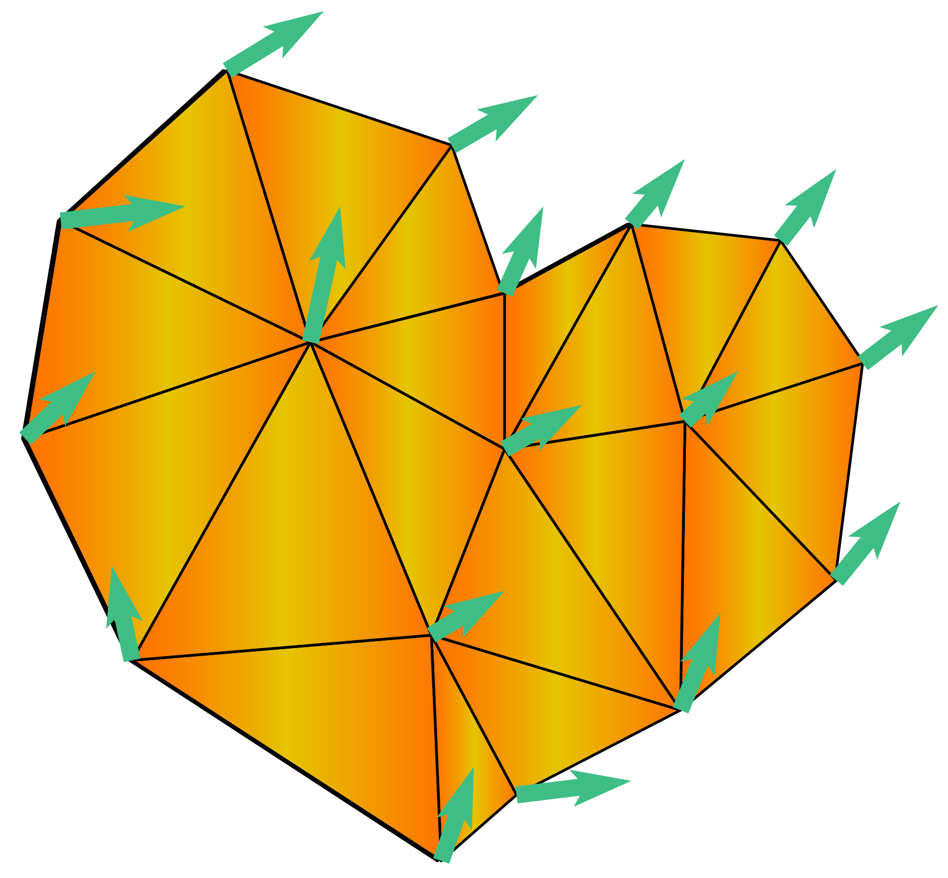
PARTICLE MPM



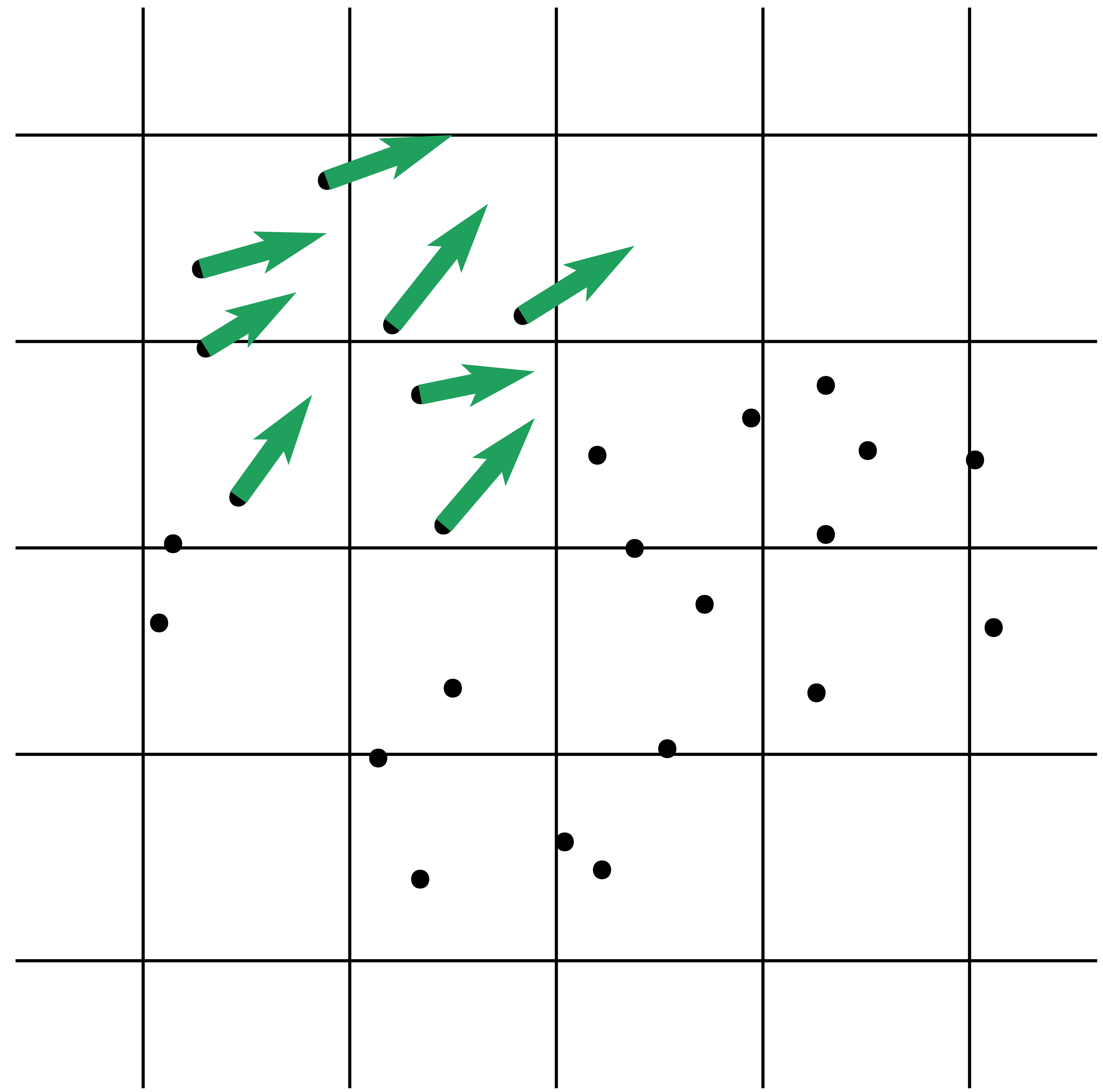


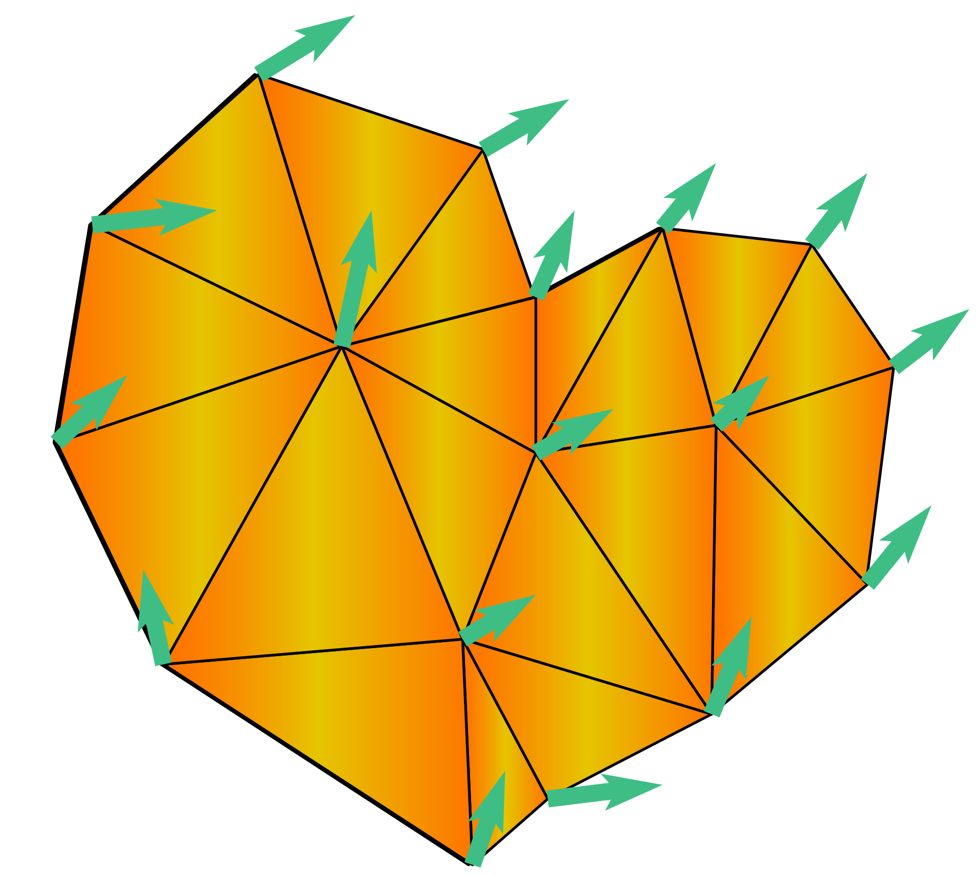
PARTICLE MPM





PARTICLE MPM





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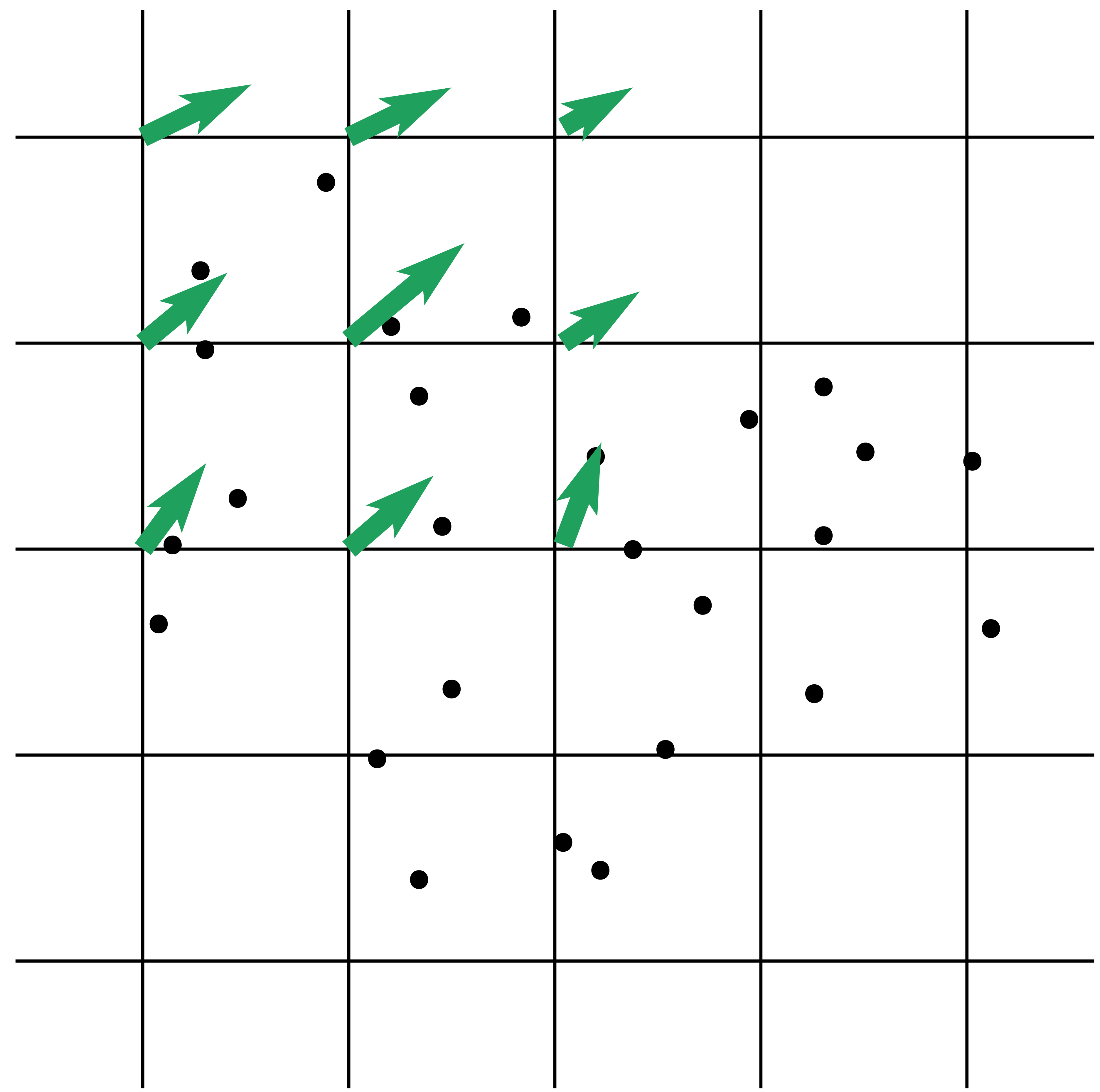
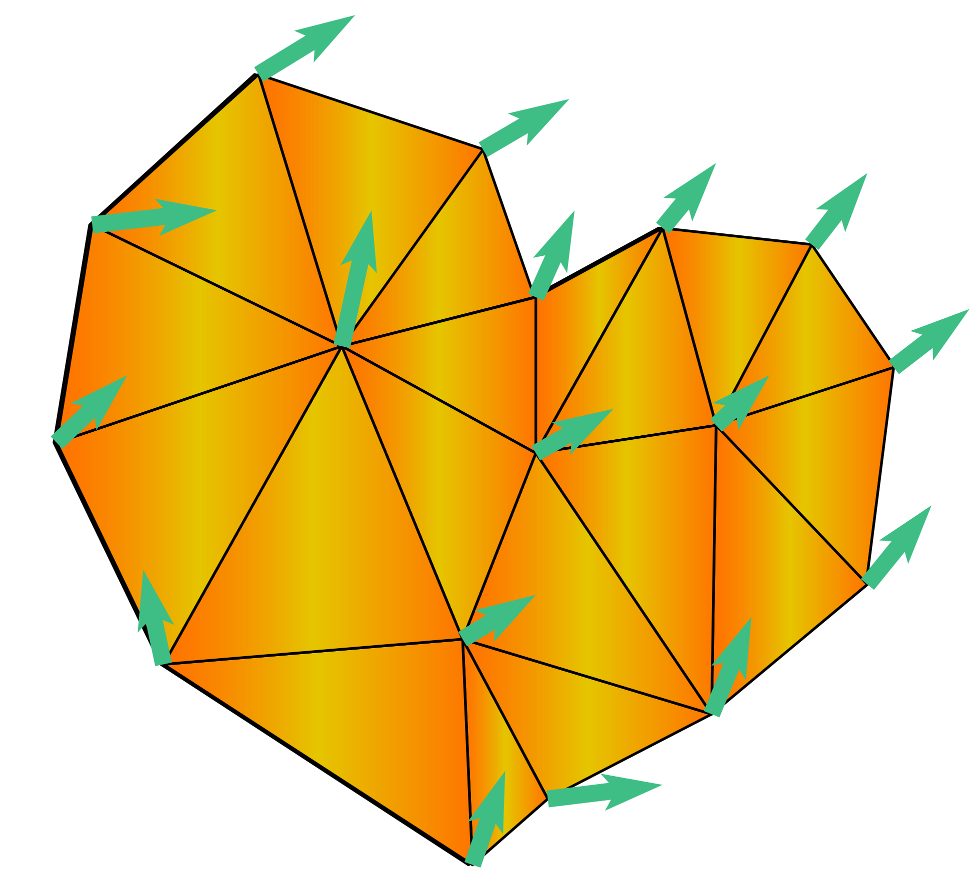
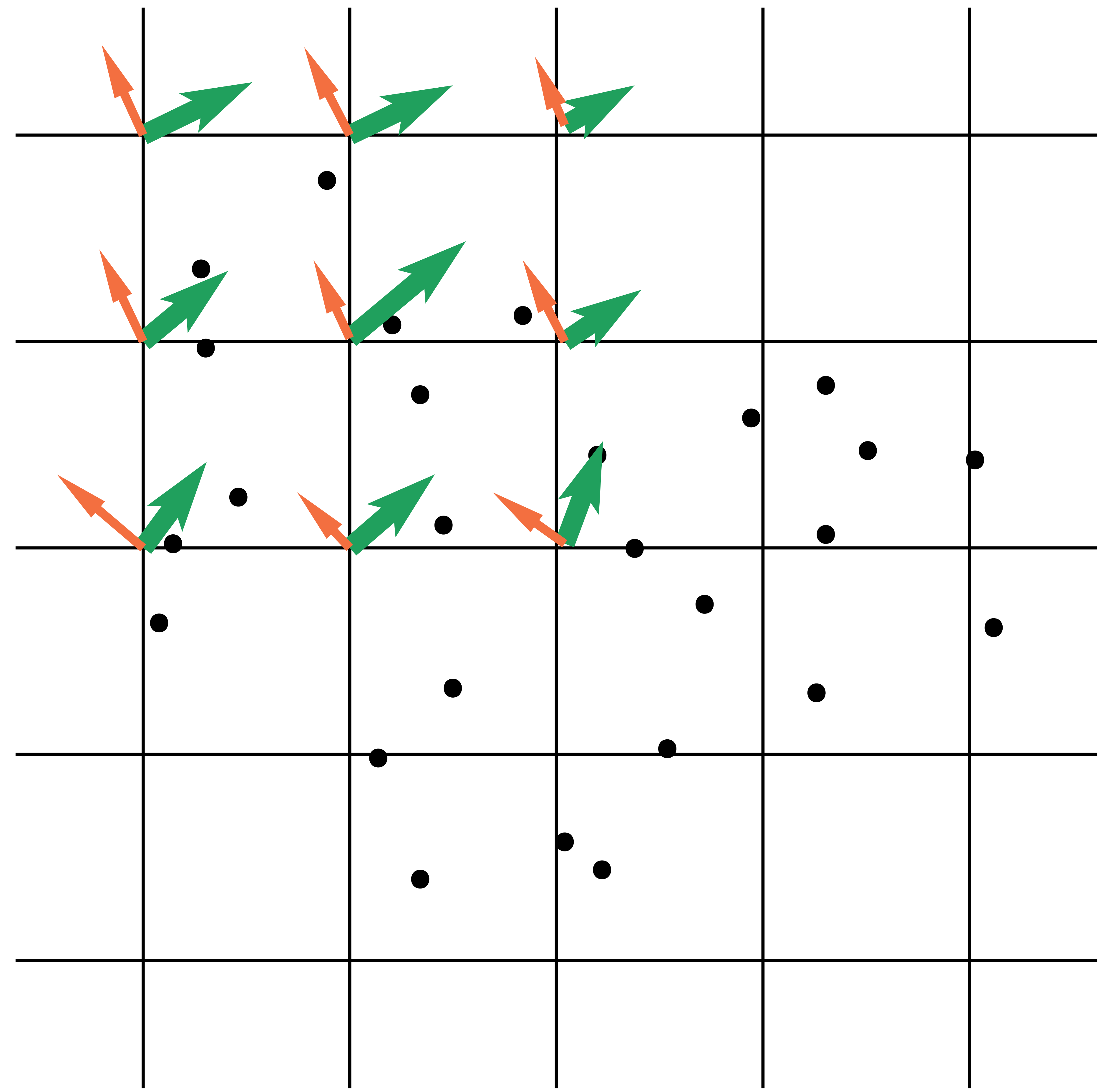
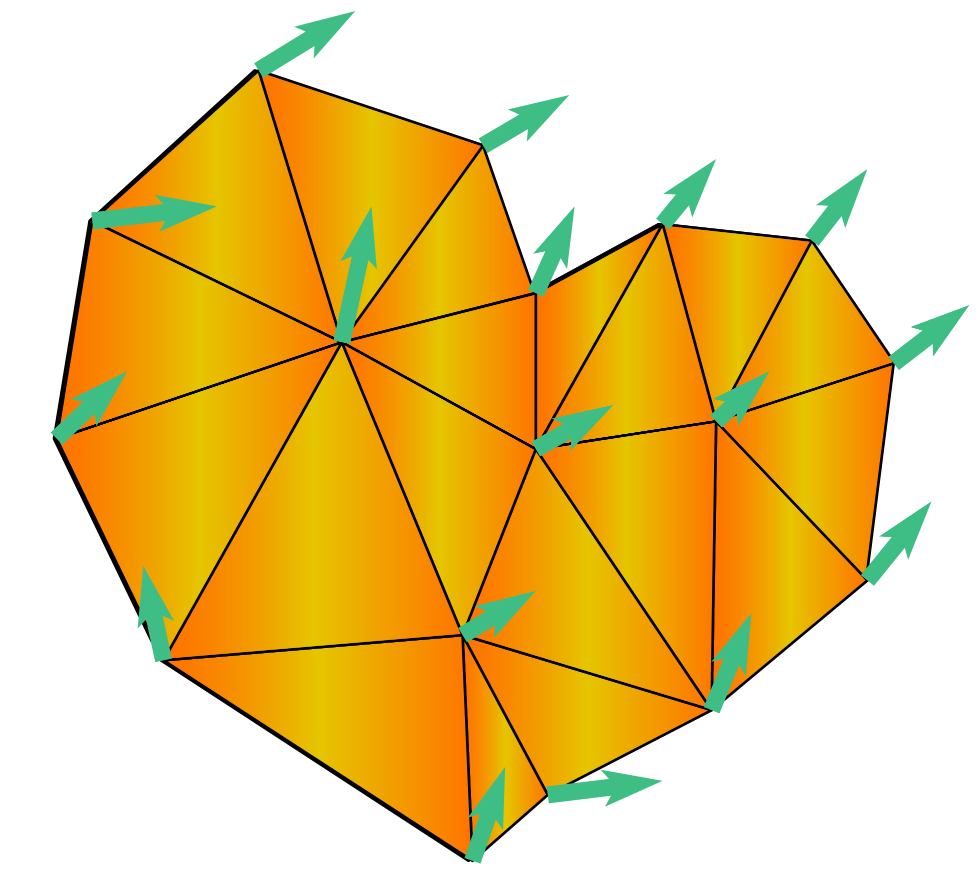


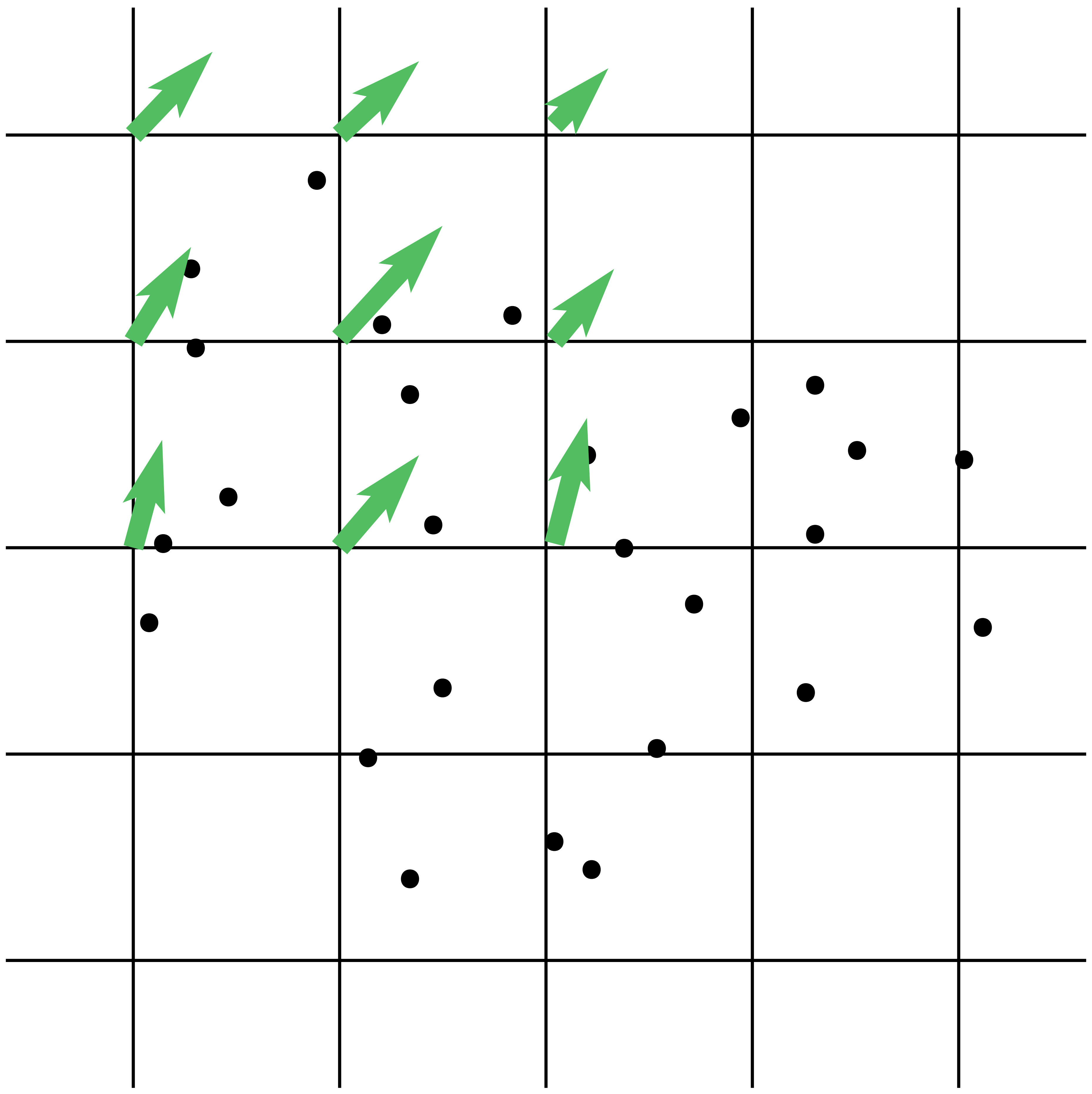


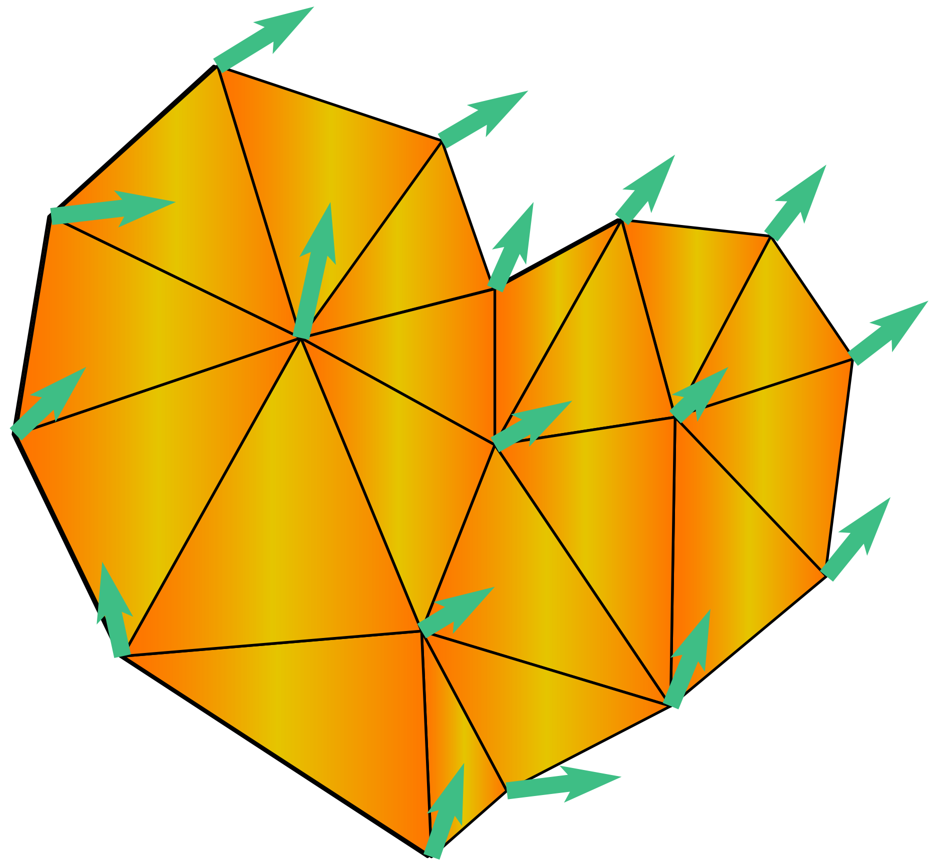
PARTICLE MPM



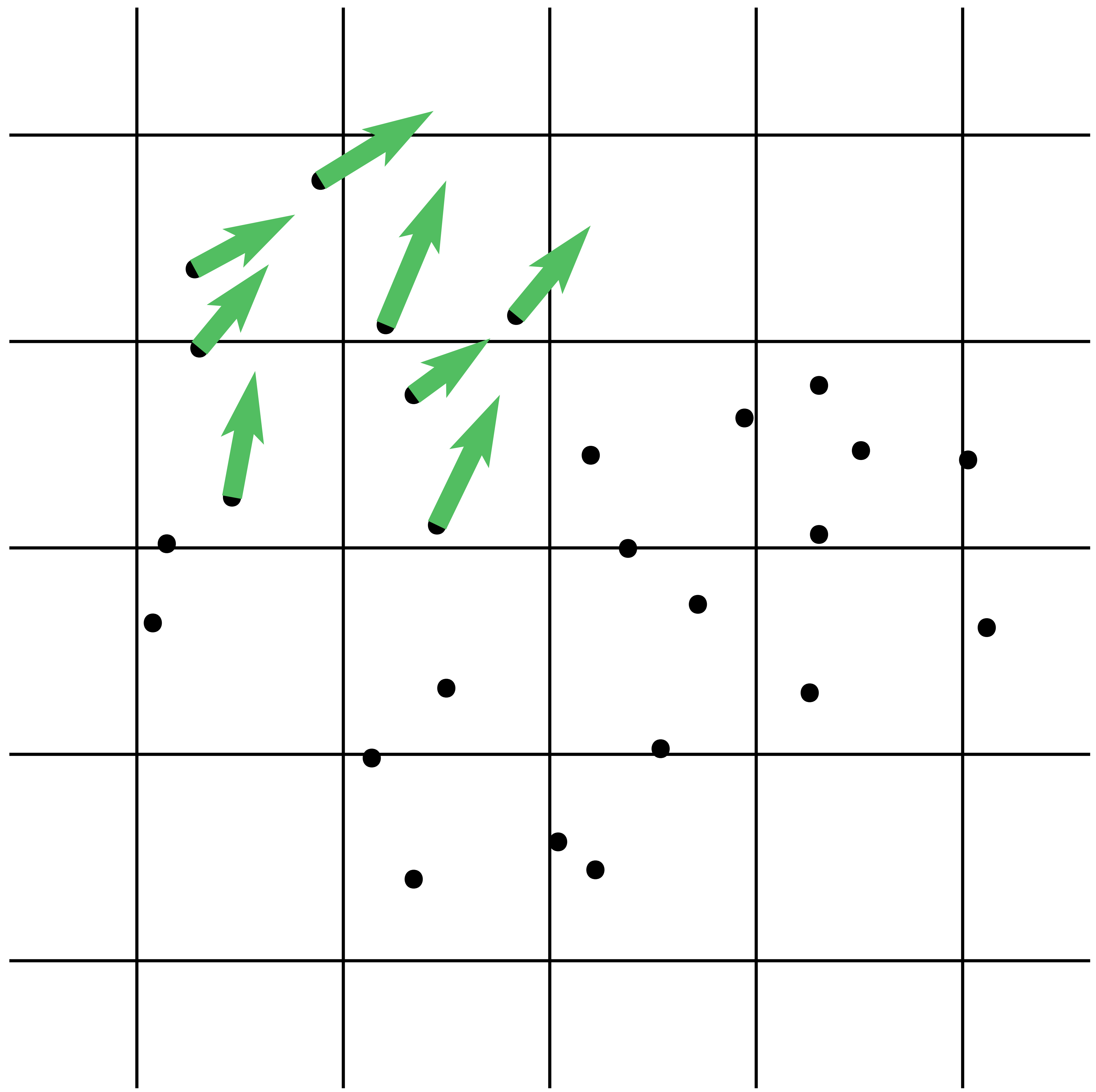


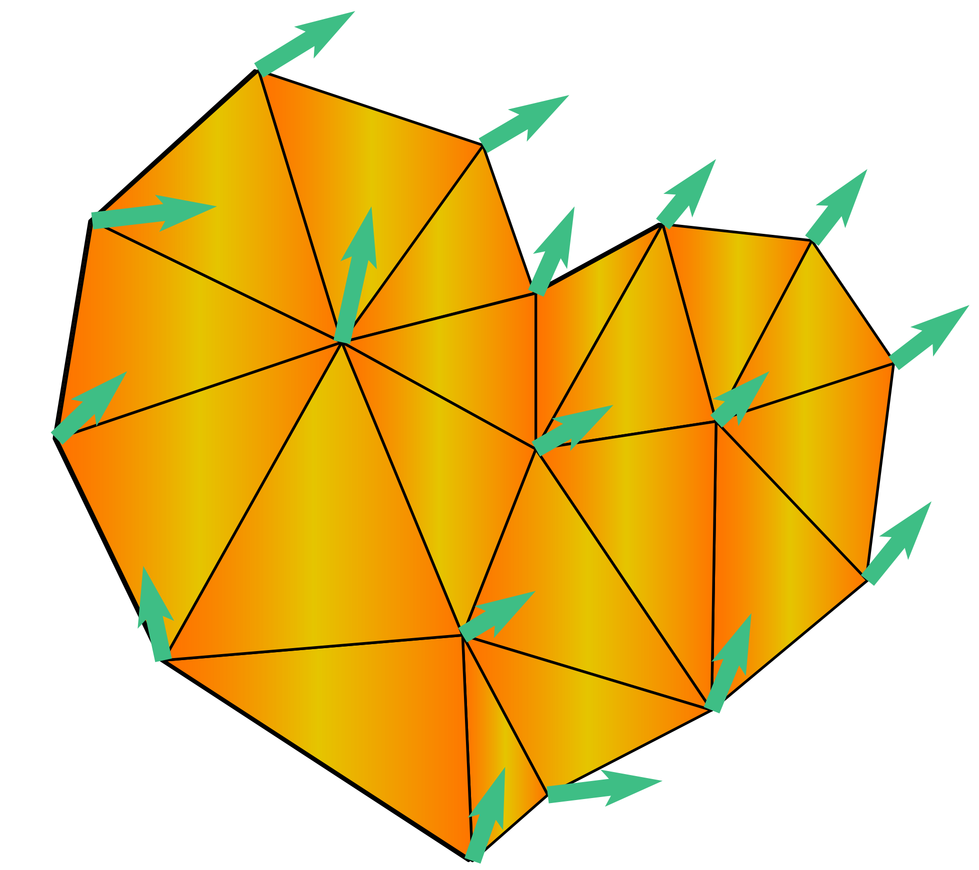
PARTICLE MPM





PARTICLE MPM

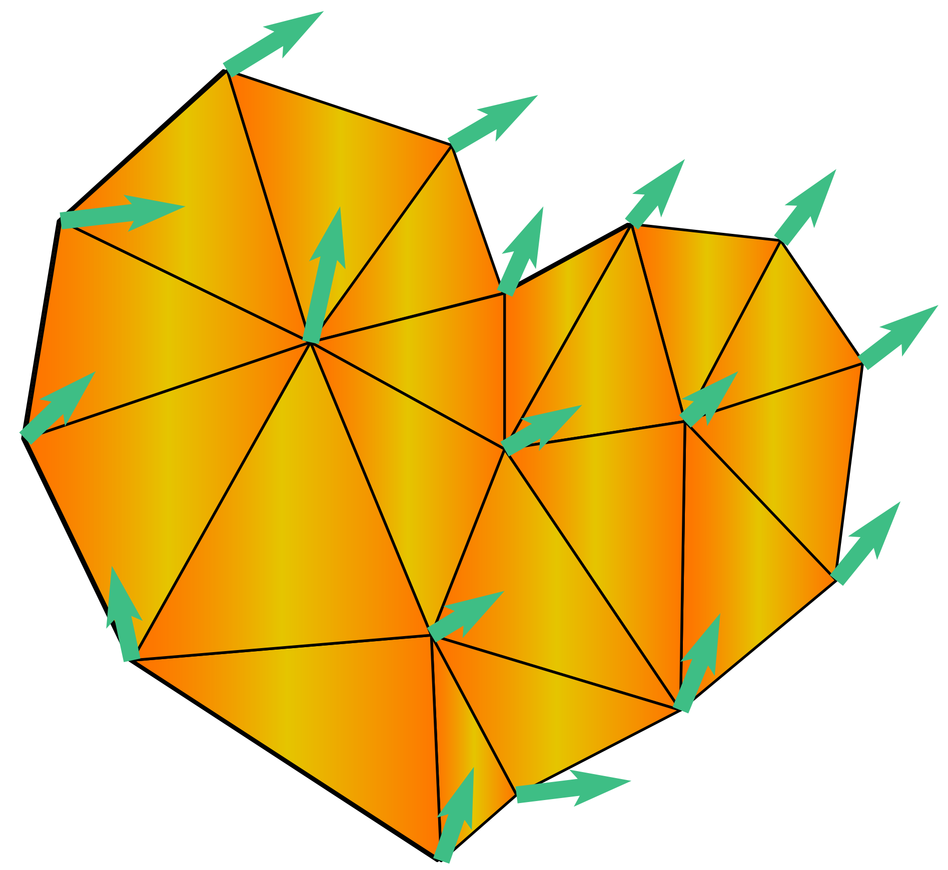




$$\Phi = \sum_p V_p^0 \Psi(\mathbf{F}_p)$$

$$\mathbf{F}_p^{n+1} = \left(\mathbf{I} + \Delta t \sum_i \mathbf{v}_i (\nabla \omega_{ip}^n)^T \right) \mathbf{F}_p^n$$

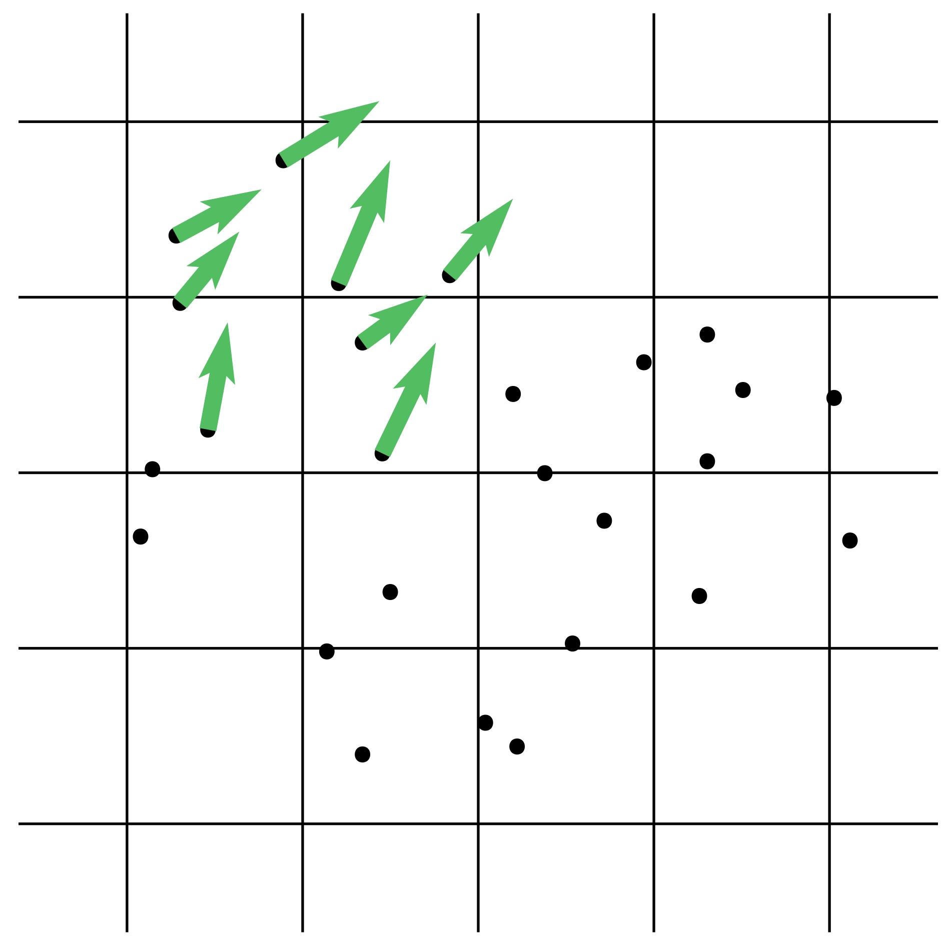
LAGRANGIAN MPM

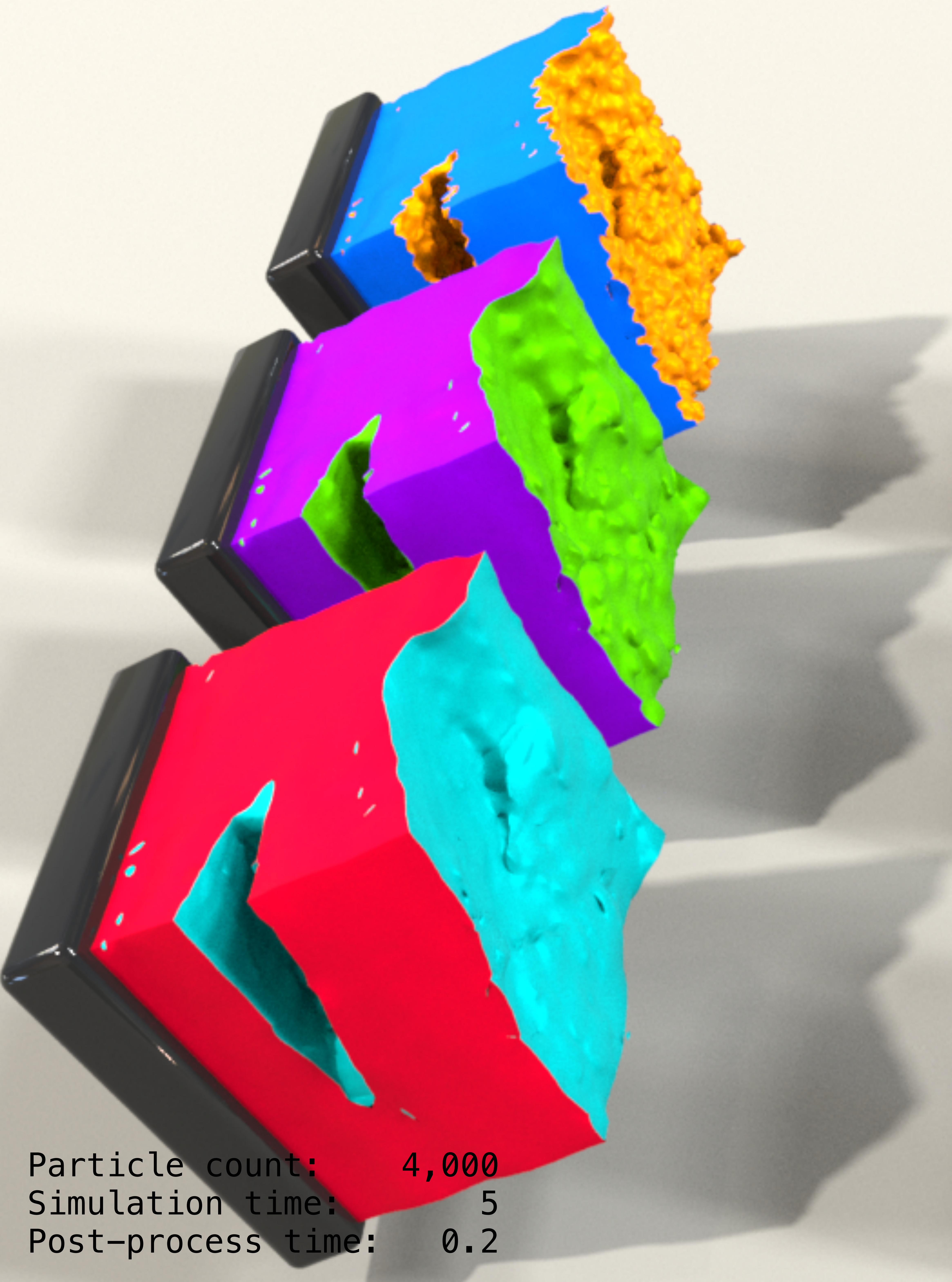


$$\Phi = \sum_e V_e^0 \Psi(\mathbf{F}_e)$$

$$\mathbf{F}_e^n = \sum_q \mathbf{x}_q^n \nabla N_q(\mathbf{X}_e)^T$$

$$\mathbf{f}_i^n = \sum_q \omega_{iq}^n \mathbf{f}_q^n$$





SIMULATION AND VISUALIZATION OF DUCTILE FRACTURE

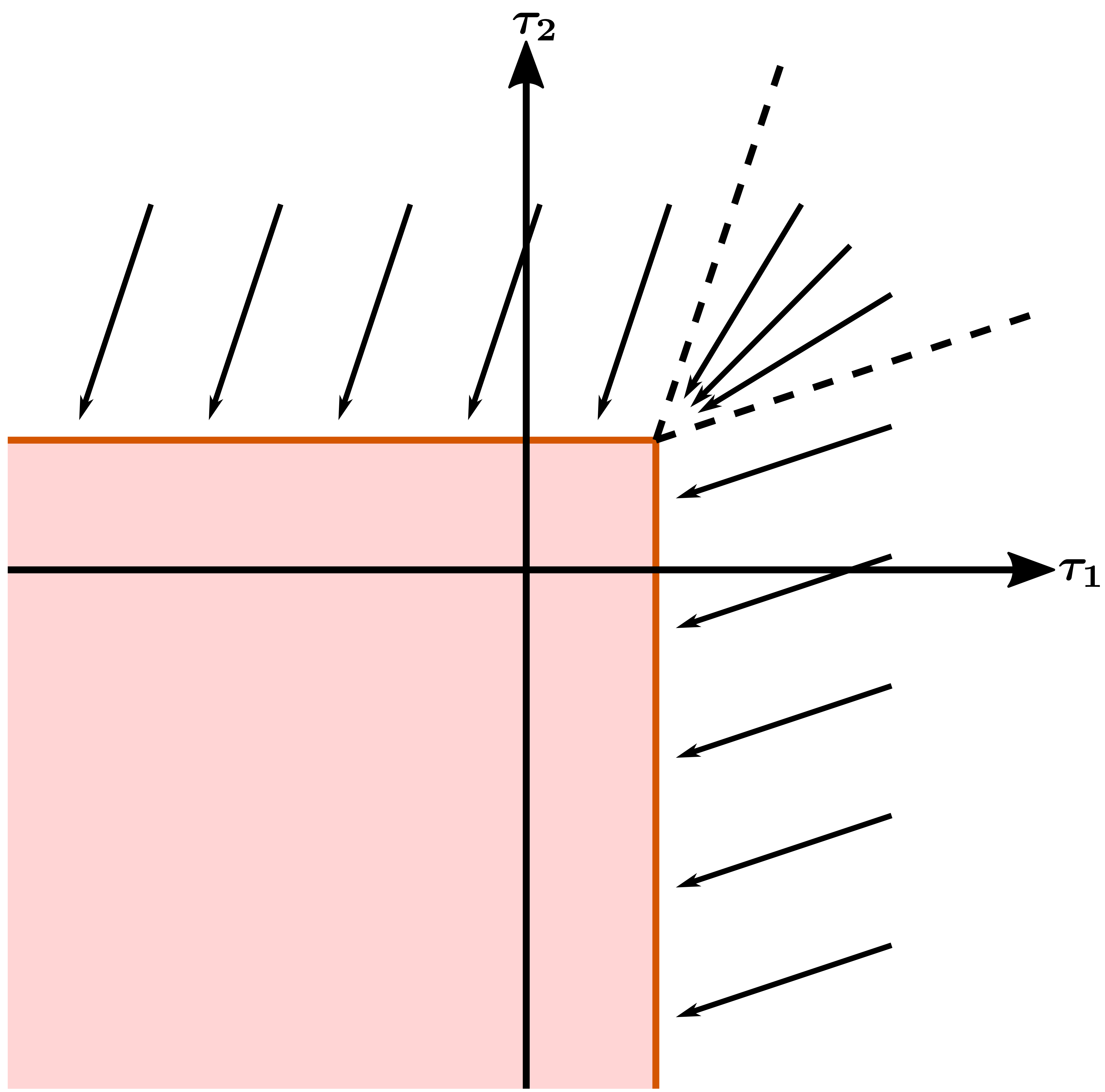
RANKINE YIELD SURFACE [MÜLLER ET AL. 2014]

- Constraining maximal principal stress

$$y(\tau) = \max_{\|\mathbf{u}\|=\|\mathbf{v}\|=1} \mathbf{u}^T \tau \mathbf{v} - \tau_C \leq 0$$

- Mode I yielding (tension)
- Softening rule

$$\tau_C^{n+1} = \tau_C^n + \alpha \left(\max_{\|\mathbf{u}\|=\|\mathbf{v}\|=1} \mathbf{u}^T \epsilon^{n+1} \mathbf{v} - \max_{\|\tilde{\mathbf{u}}\|=\|\tilde{\mathbf{v}}\|=1} \tilde{\mathbf{u}}^T \epsilon^{tr} \tilde{\mathbf{v}} \right)$$



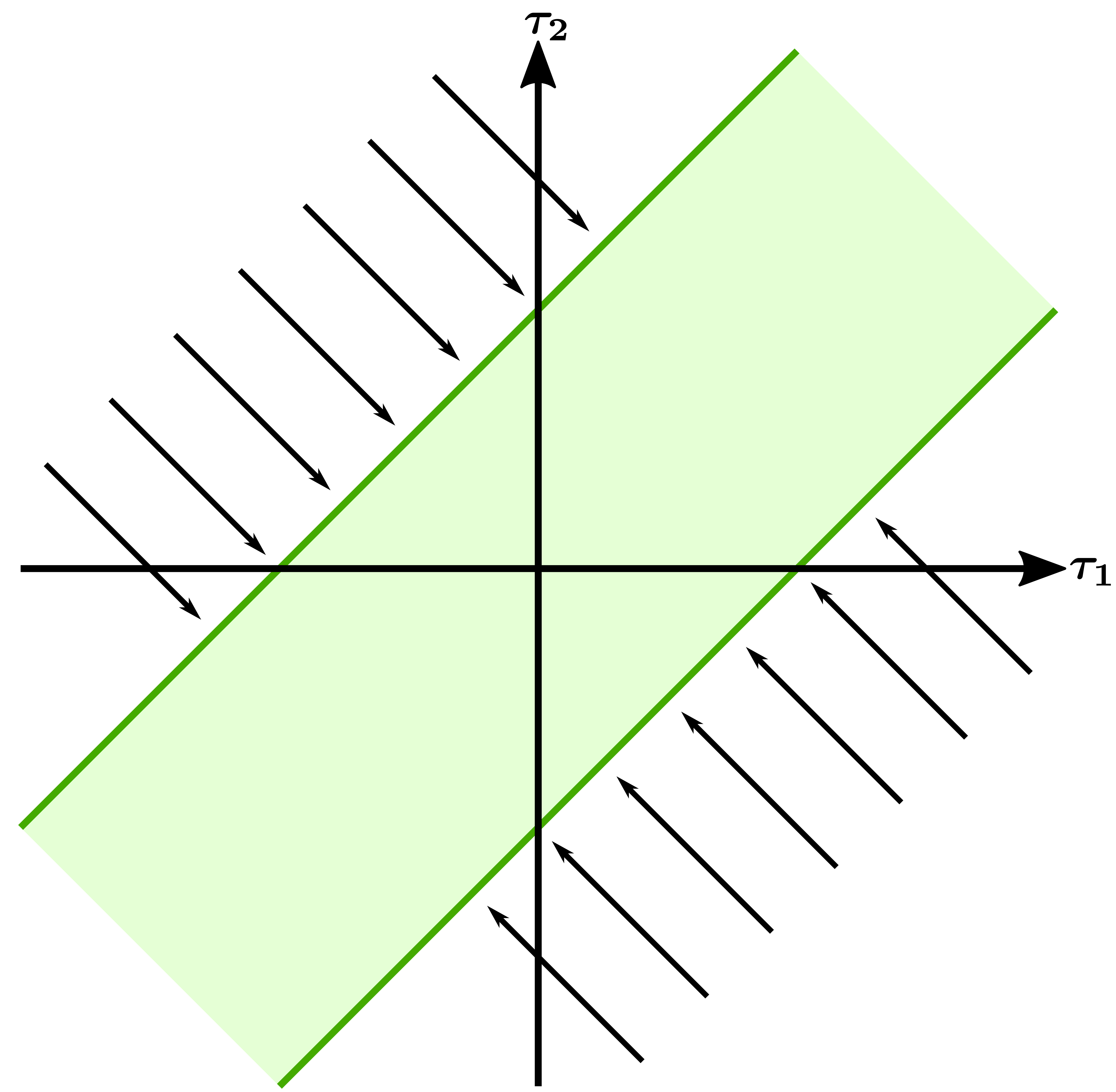

Particle count: 130,000
Simulation time: 15
Post-process time: 8

VON MISES (J2) YIELD SURFACE

- Constraining shear stress

$$y(\tau) = \|\tau - \mathrm{tr}(\tau)\mathbf{I}\|_F - \tau_C \leq 0$$

- Mode II and III yielding (shear)
- Softening can be added



$\tau_C/E = 1$
$\tau_C/E = 0.7$
$\tau_C/E = 0.5$
Particle count: 60,000
Simulation time: 11
Post-process time: 4

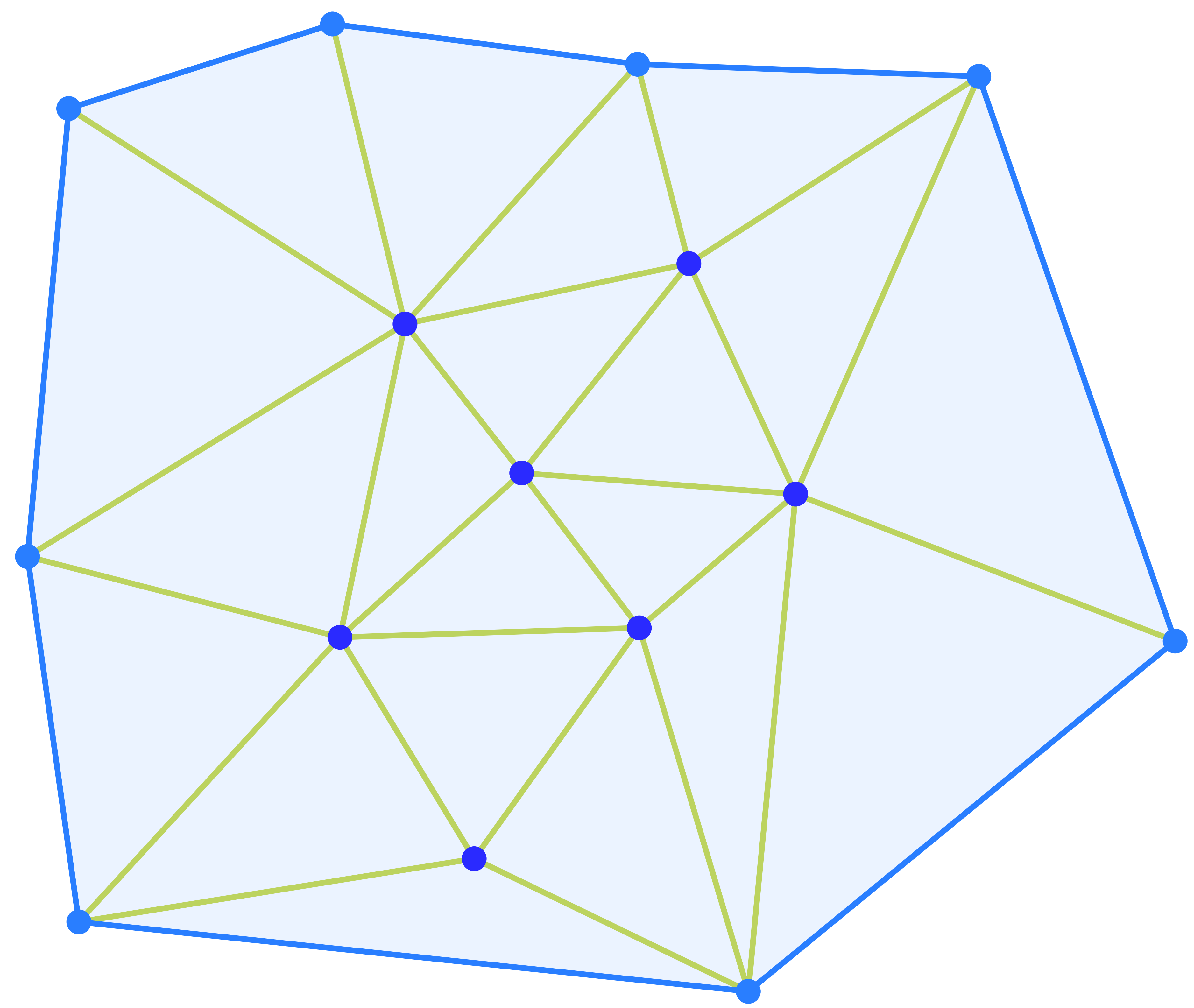
Particle count: 60,000
Simulation time: 11
Post-process time: 5

THREE STEPS OF CREATING FRACTURING MESH

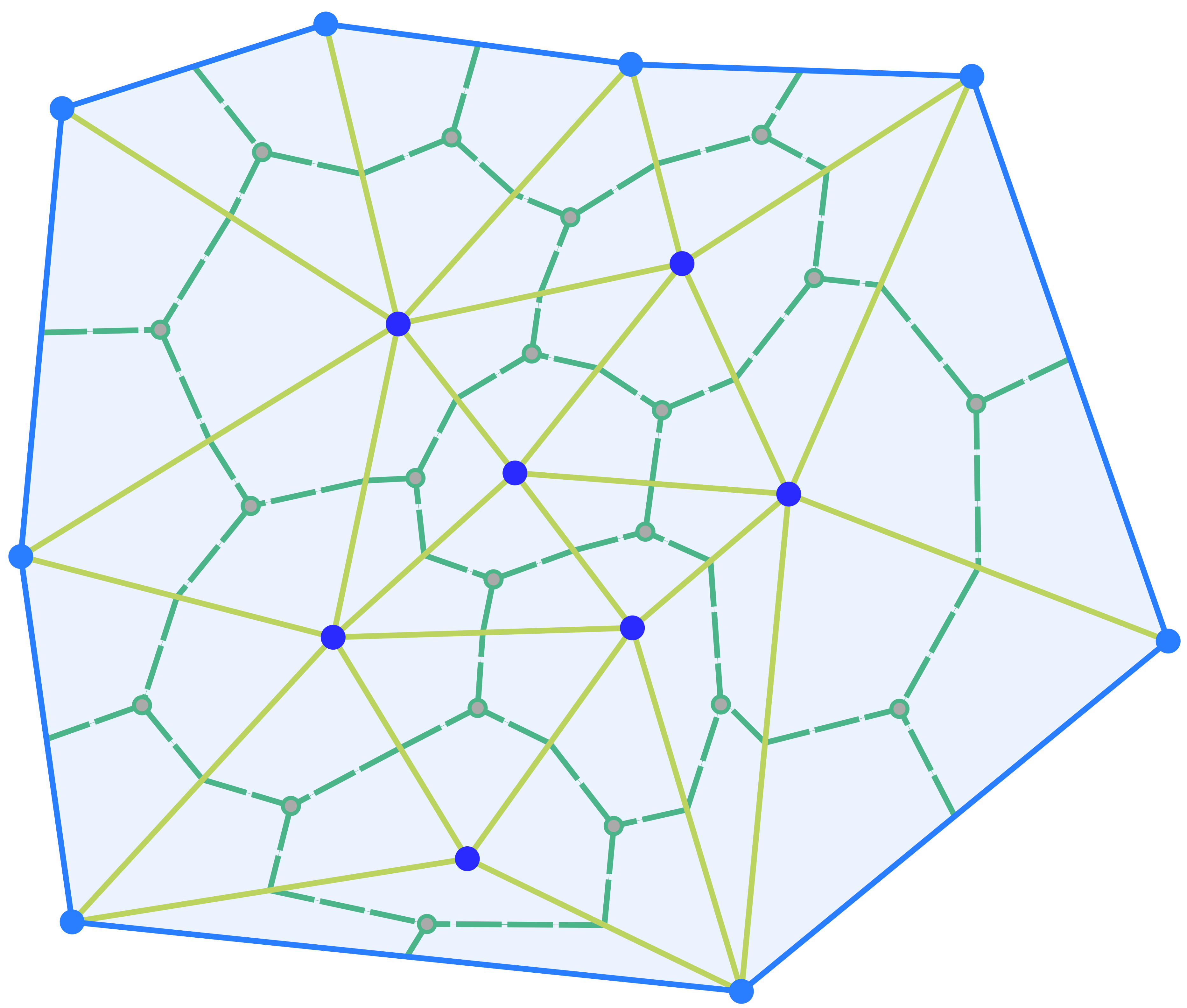
- Fracturing topology (that evolves with time)
- Extrapolate positions for the added vertices
- Smoothing crack surface to reduce mesh-dependent noise
- Advantage: per-frame post-process instead of per-time-step treatment

FRACTURING TOPOLOGY

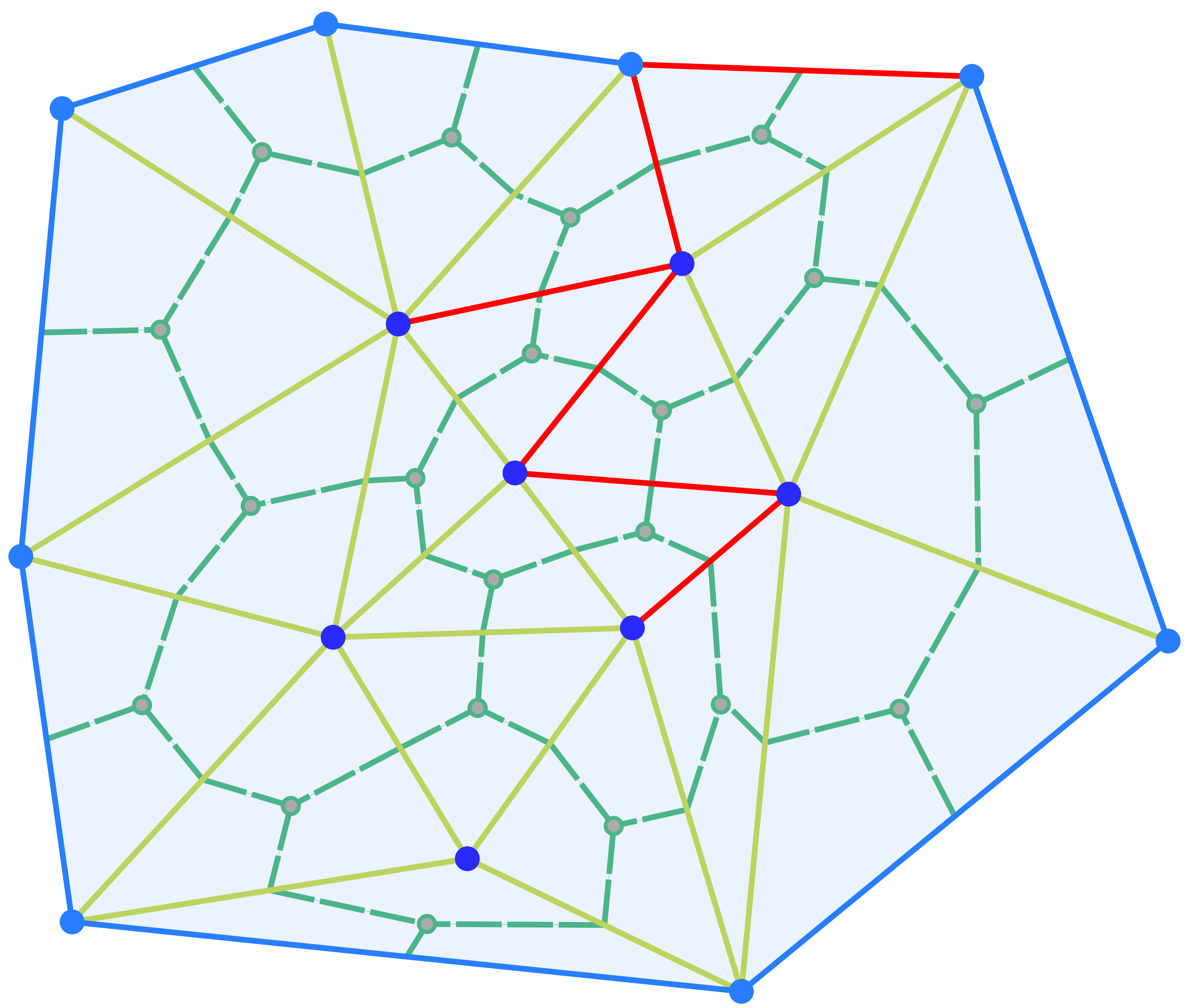
FRACTURING TOPOLOGY



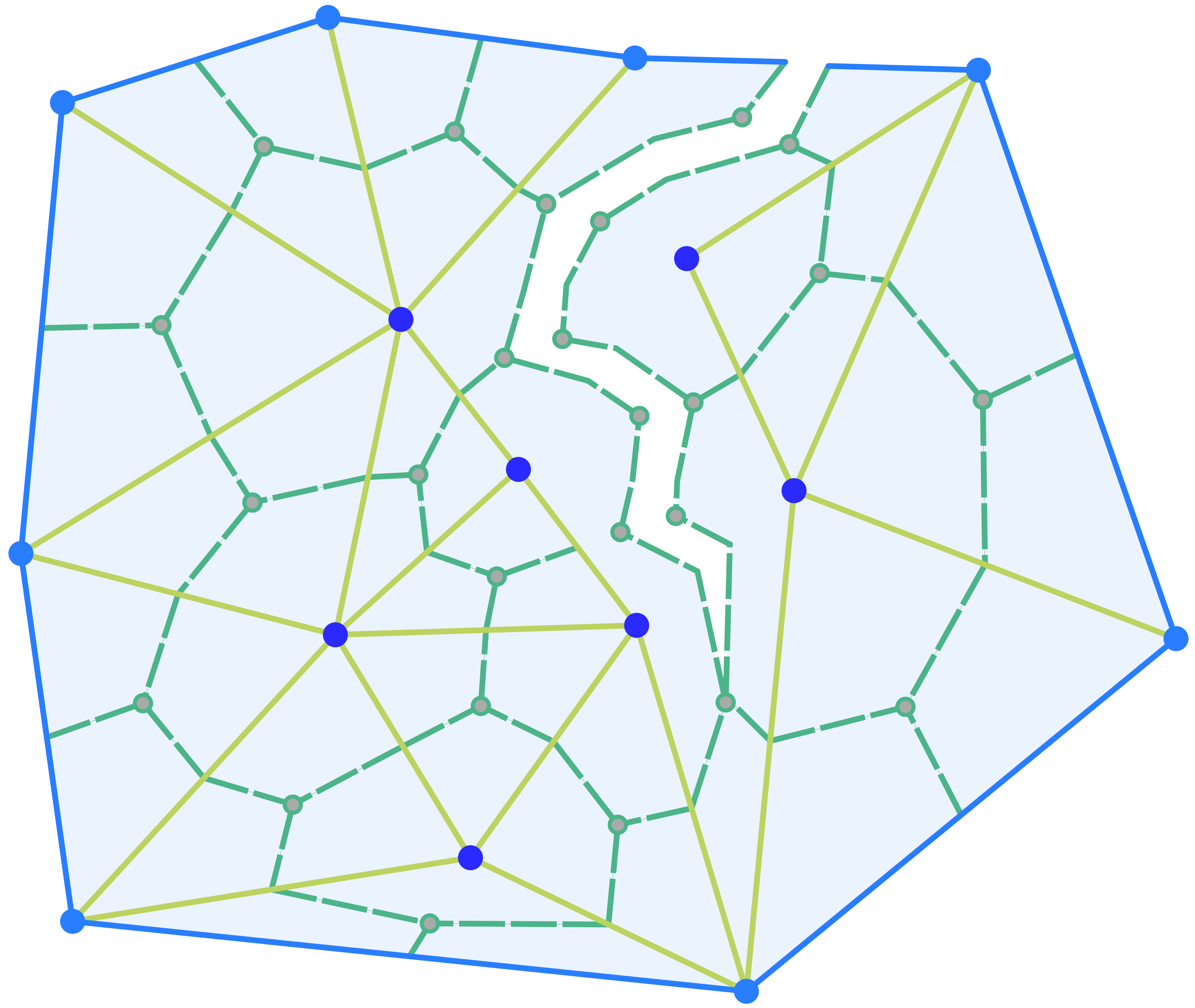
FRACTURING TOPOLOGY



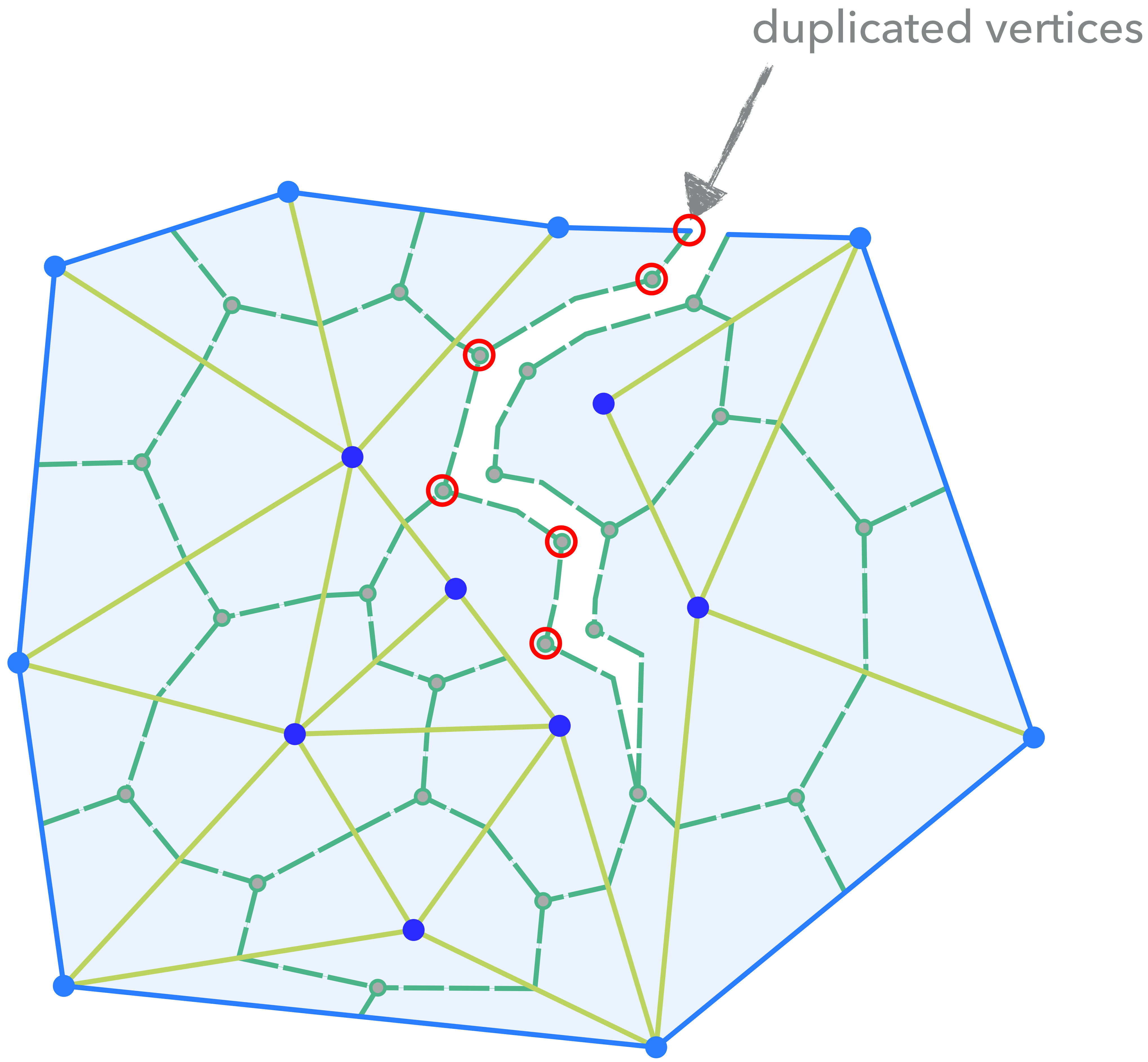
FRACTURING TOPOLOGY



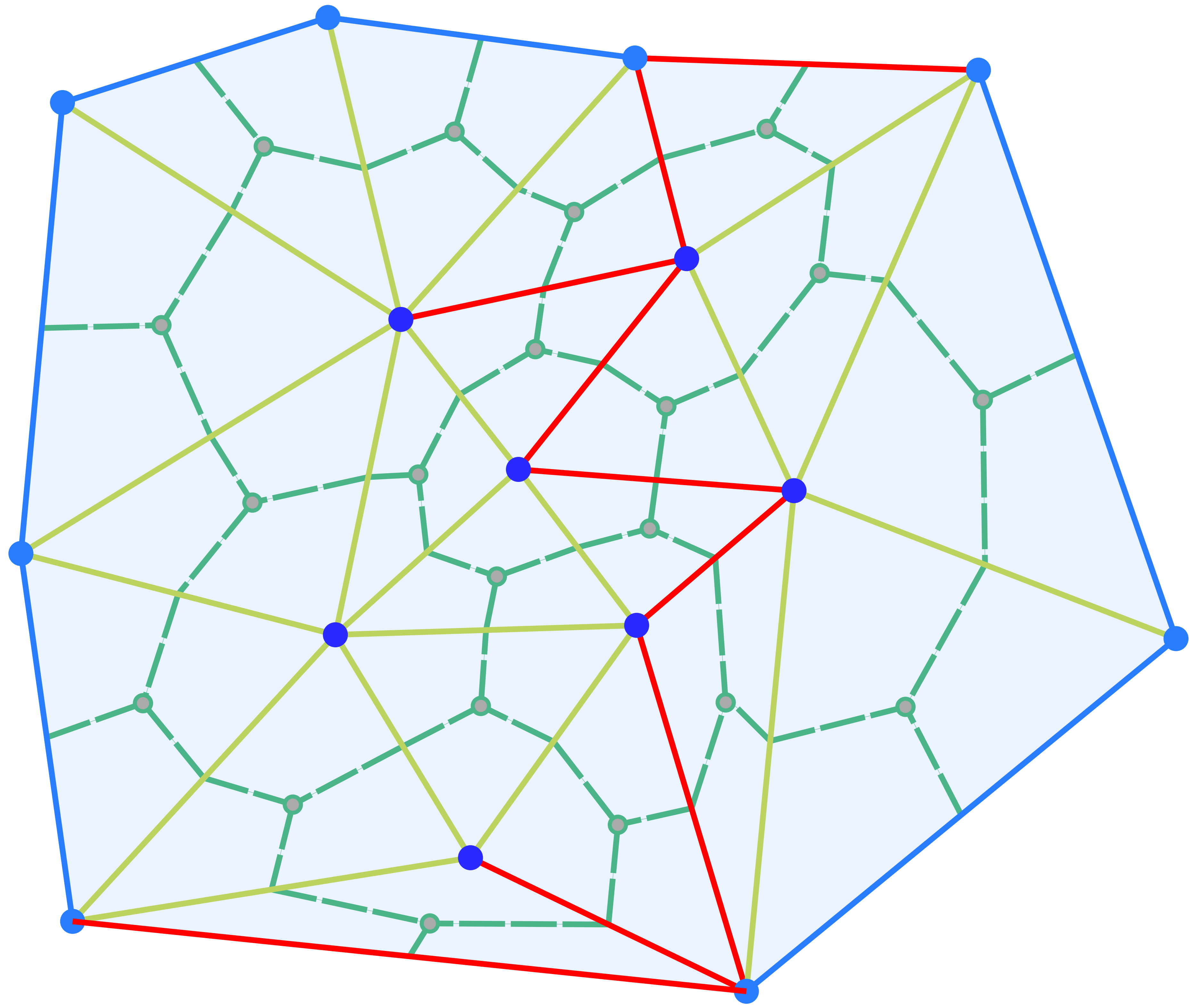
FRACTURING TOPOLOGY



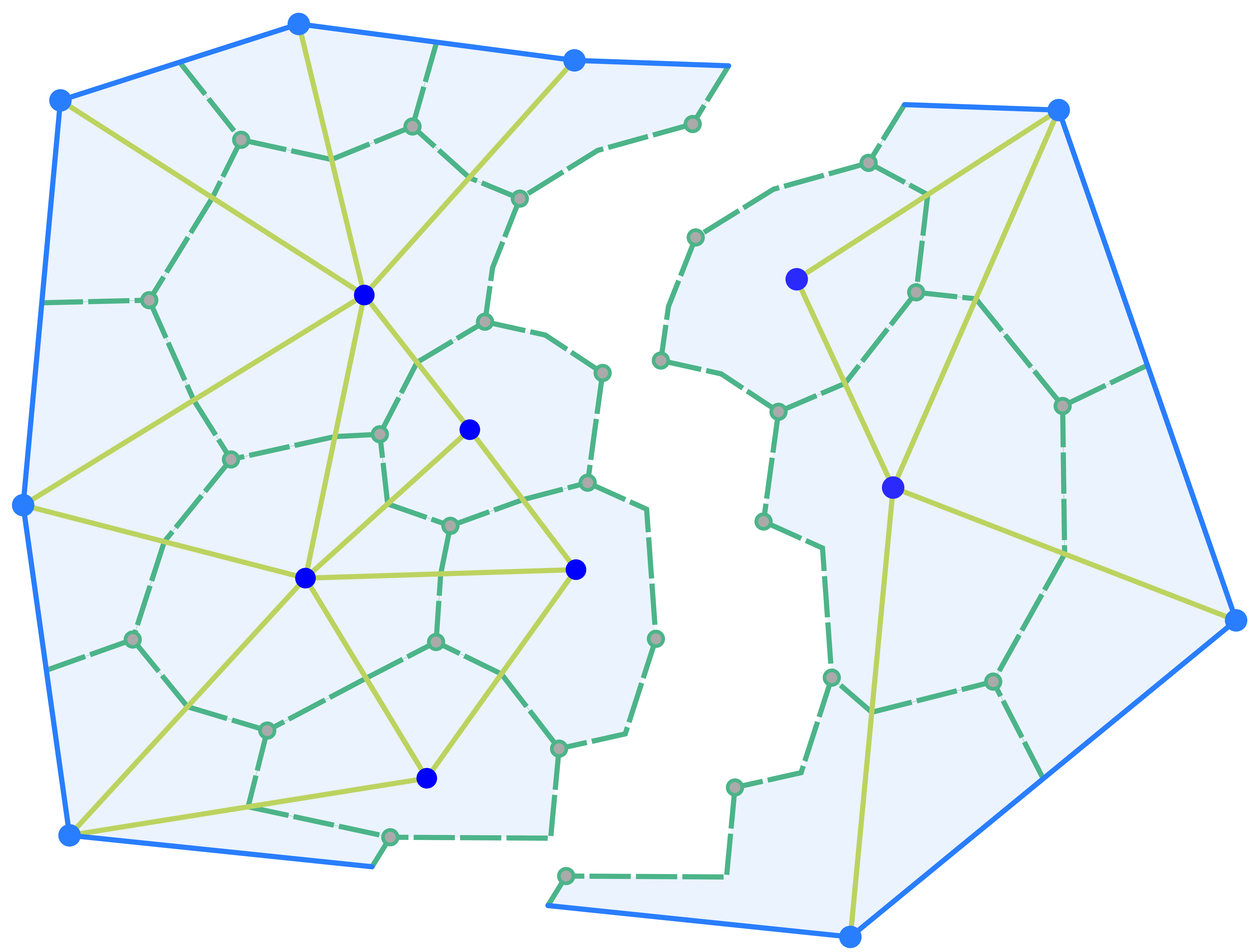
FRACTURING TOPOLOGY



FRACTURING TOPOLOGY

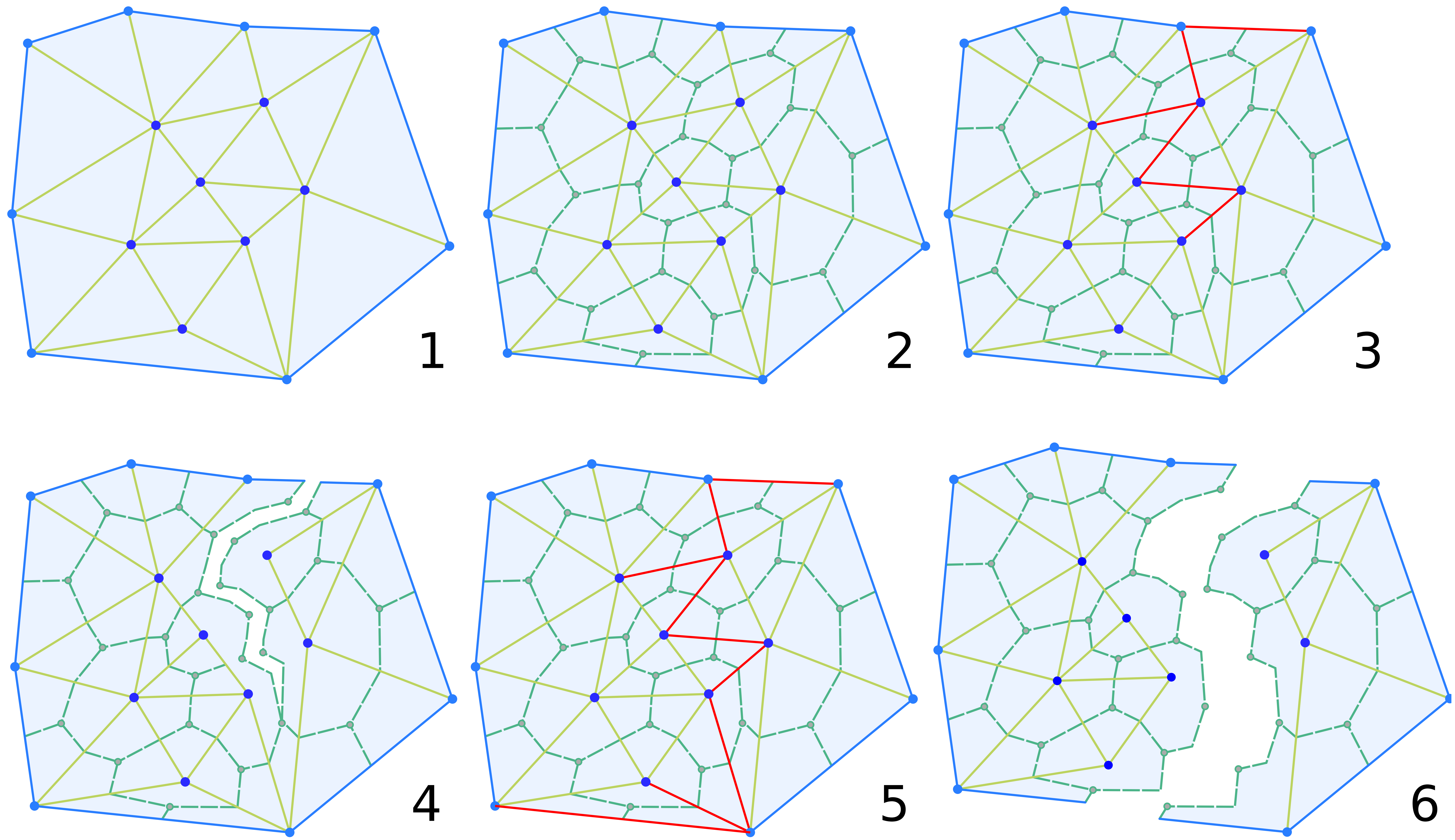


FRACTURING TOPOLOGY



FRACTURING TOPOLOGY

- Subdivided mesh
- Edge-stretching cutting criterion
- Evolves with time



EXTRAPOLATING POSITIONS FOR ADDED VERTICES

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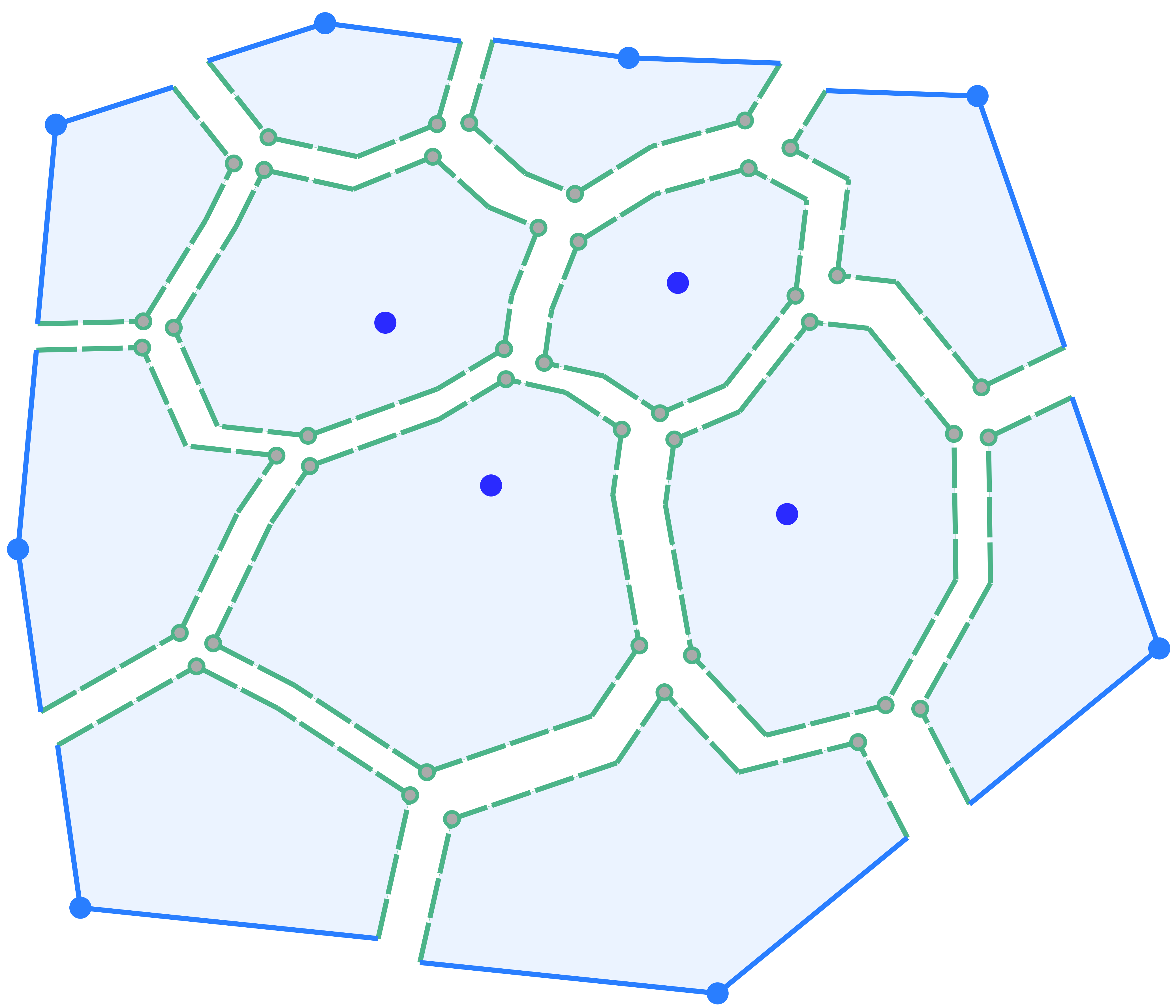

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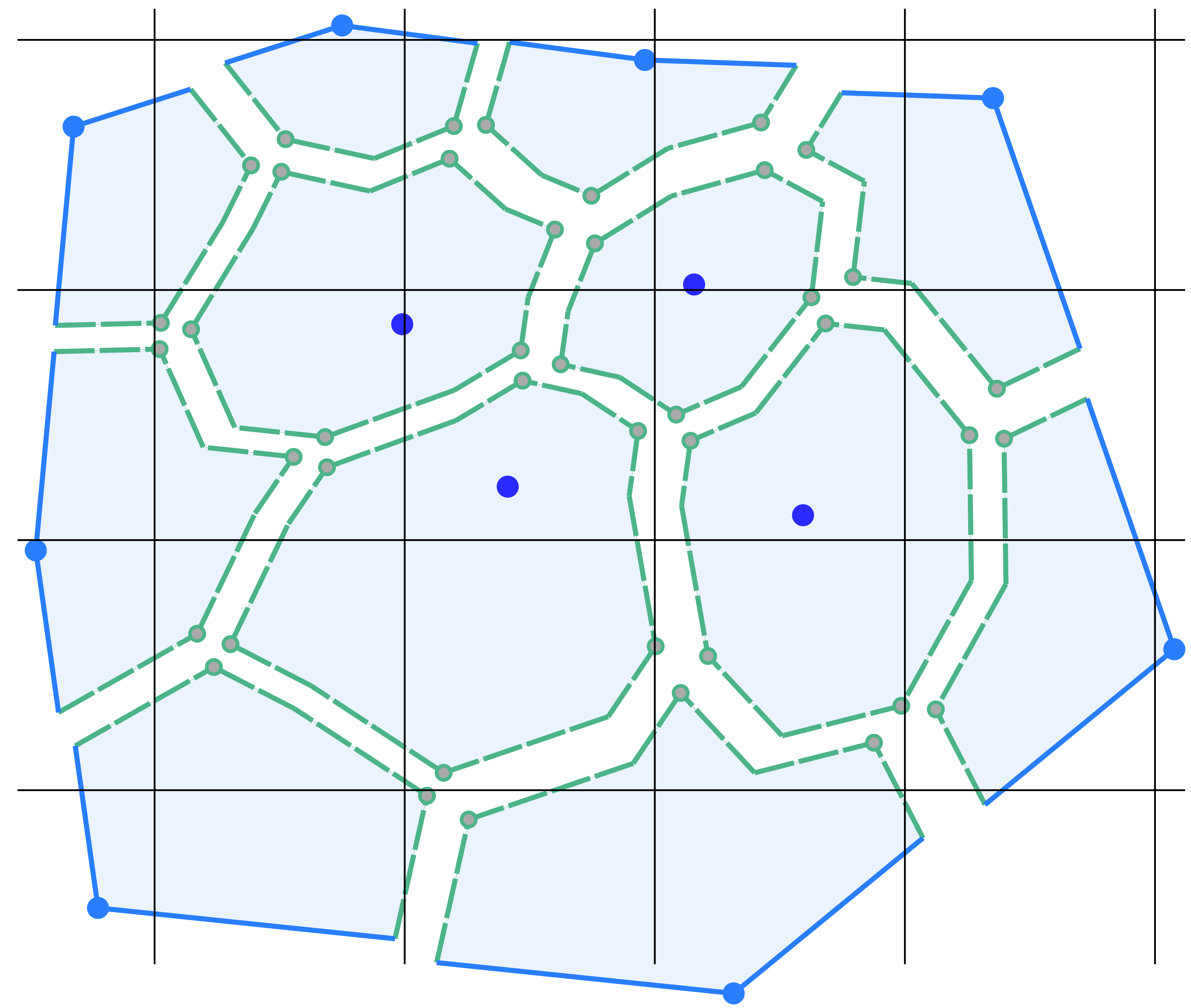

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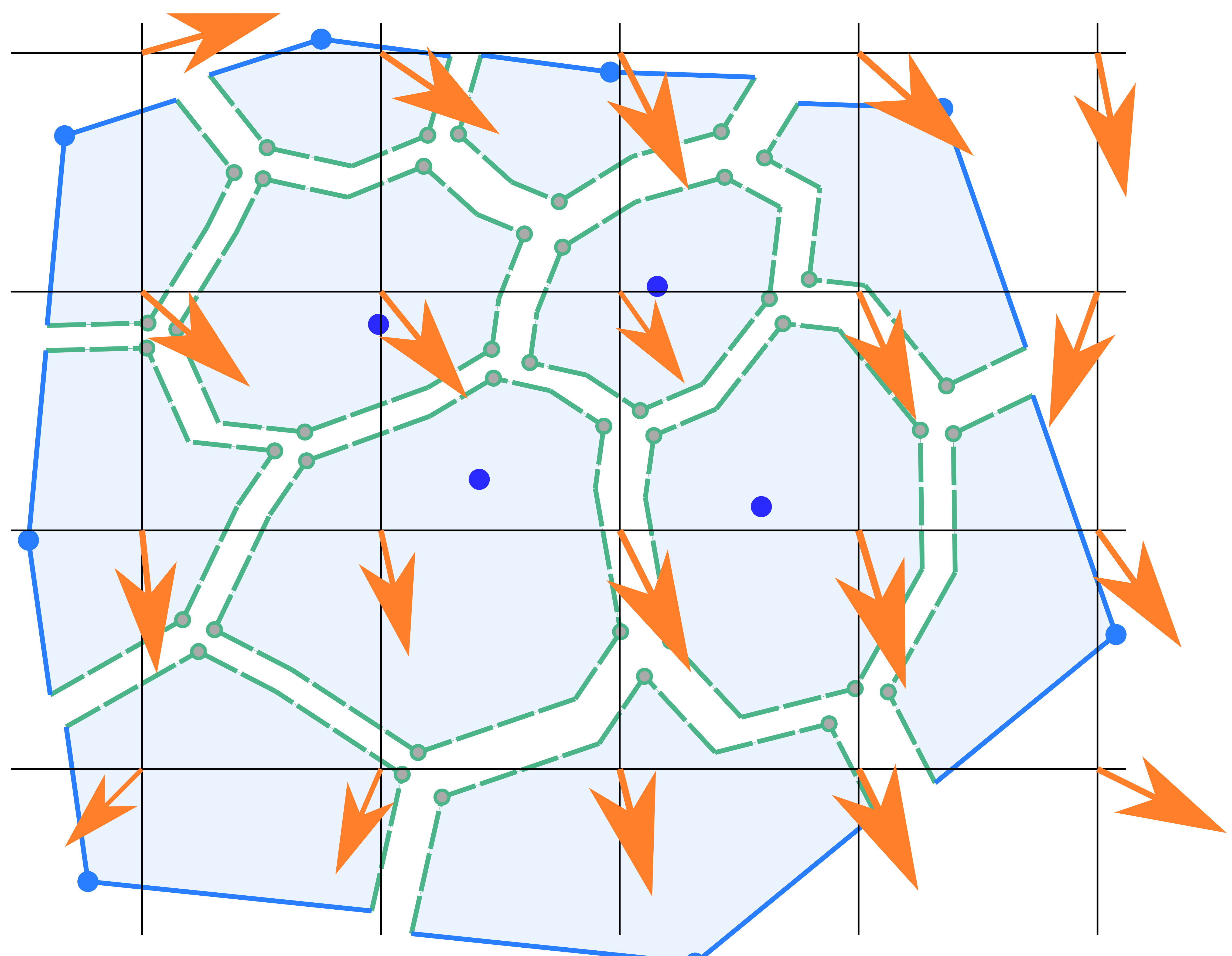
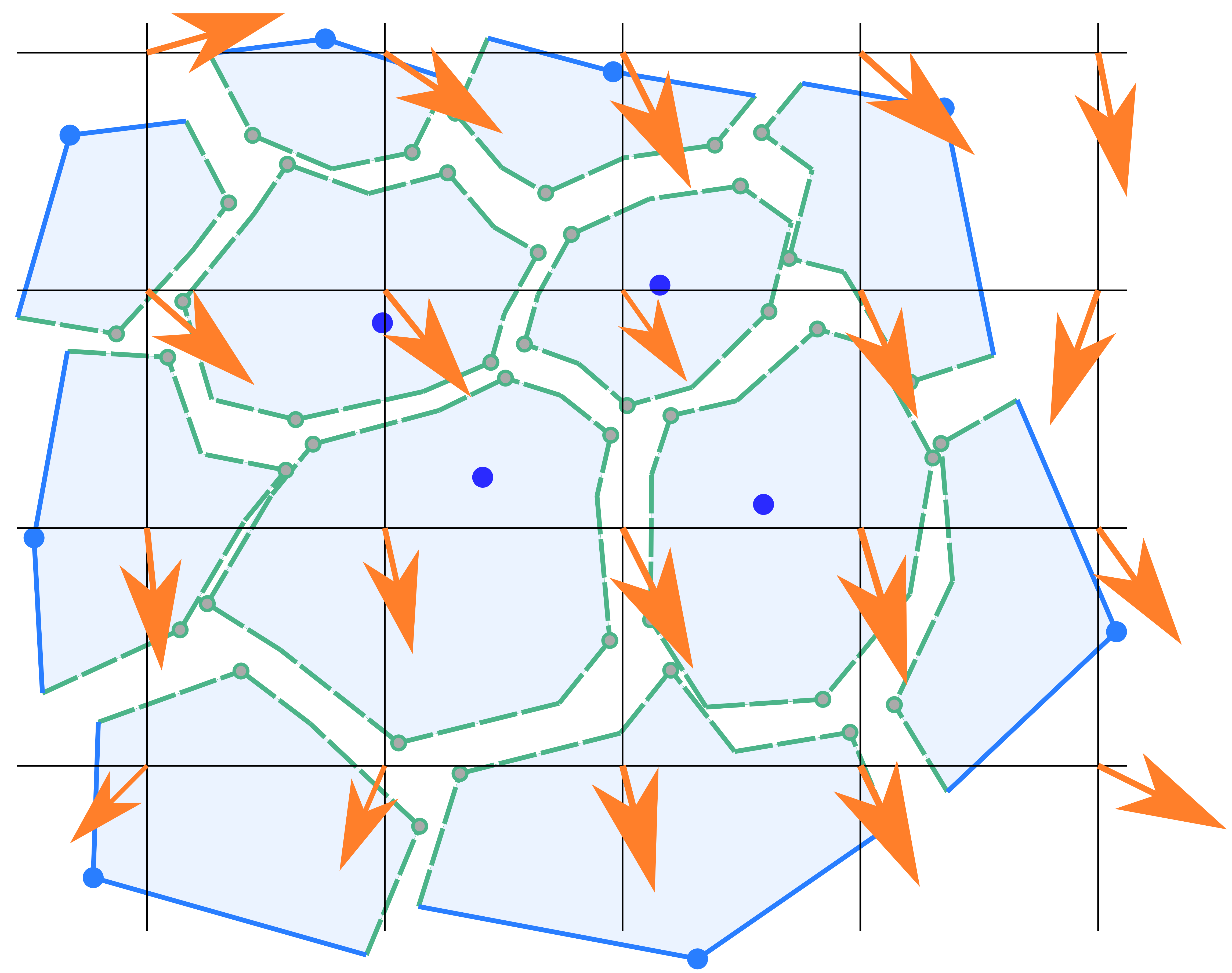
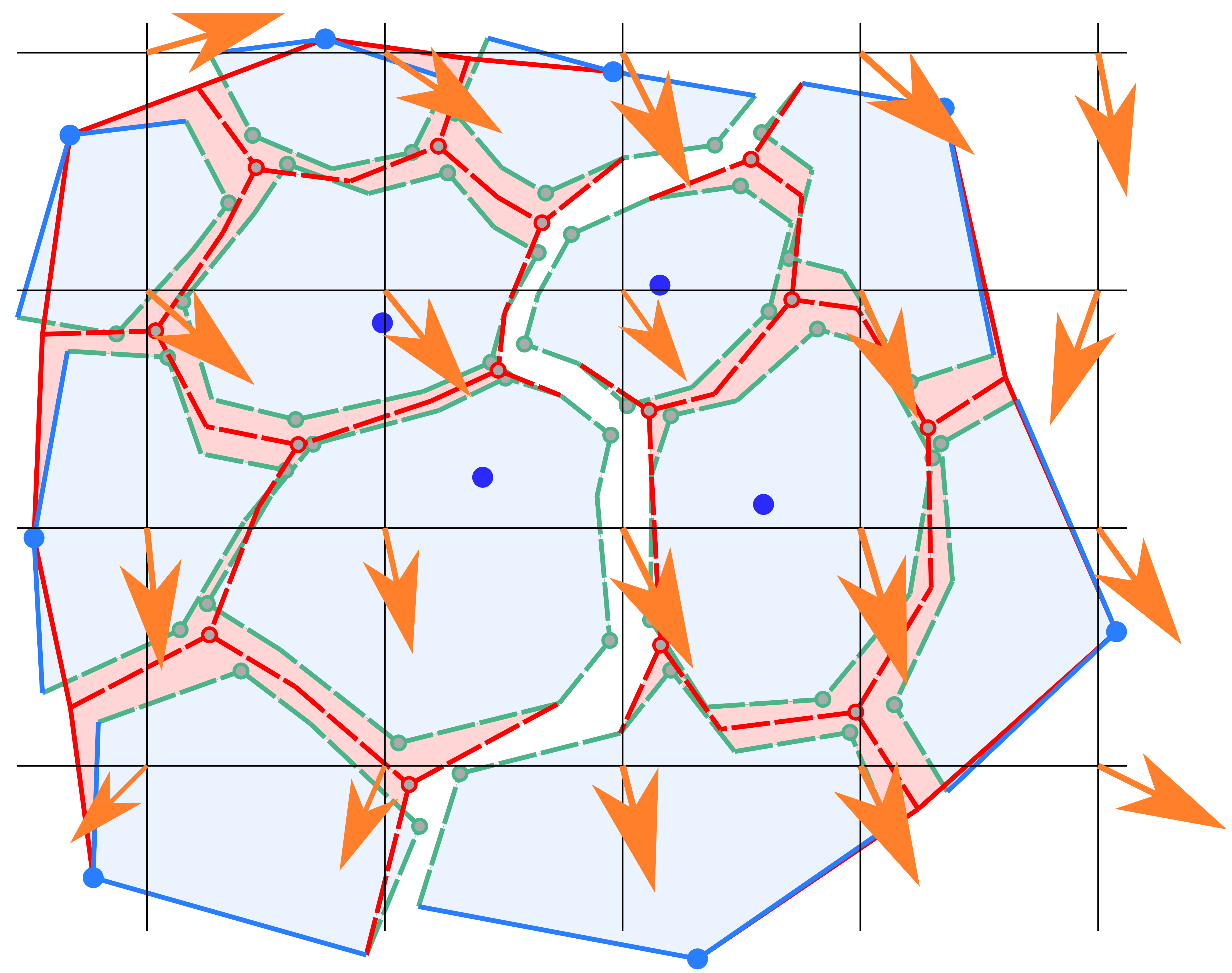


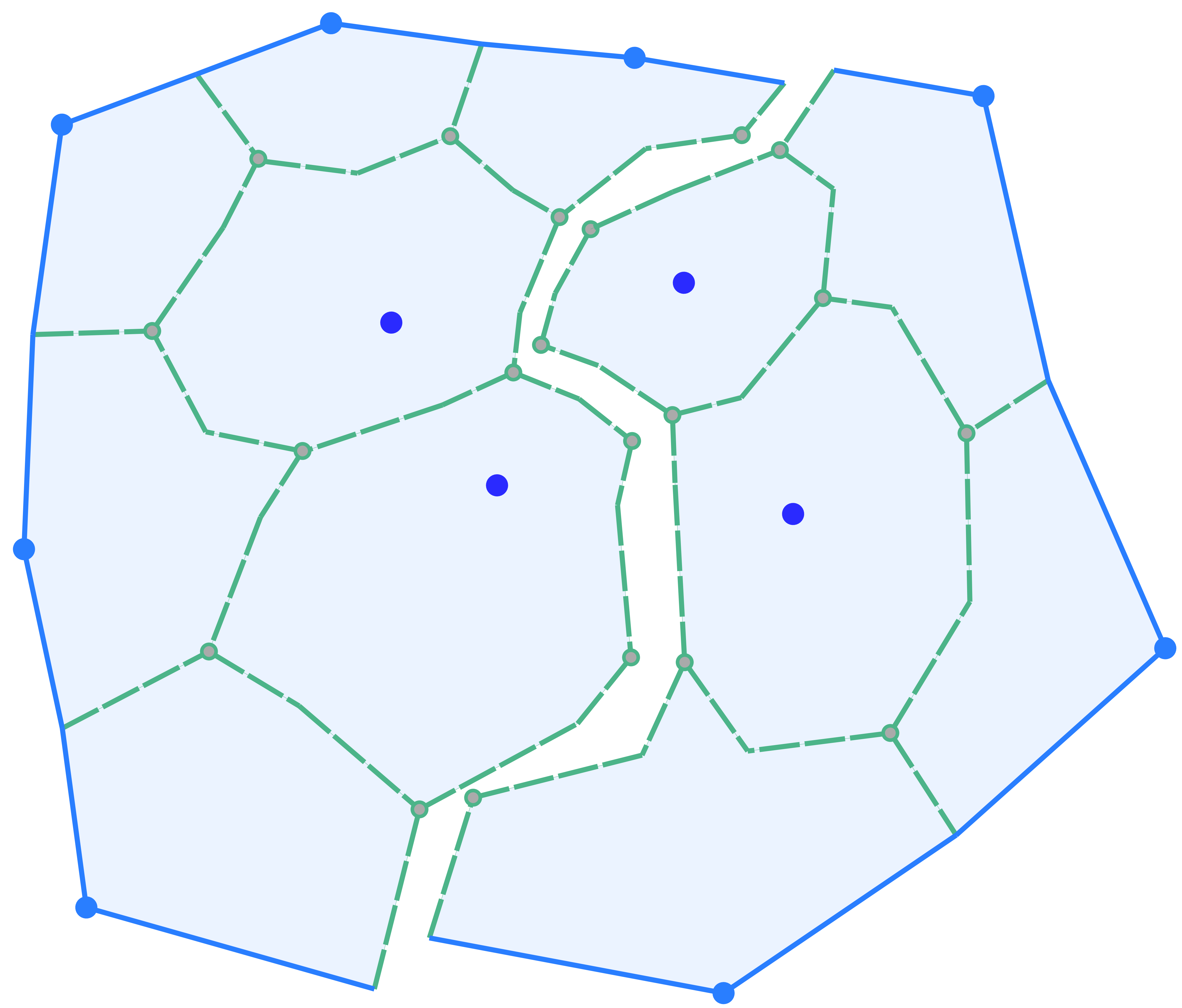
EXTRAPOLATING POSITIONS FOR ADDED VERTICES



EXTRAPOLATING POSITIONS FOR ADDED VERTICES

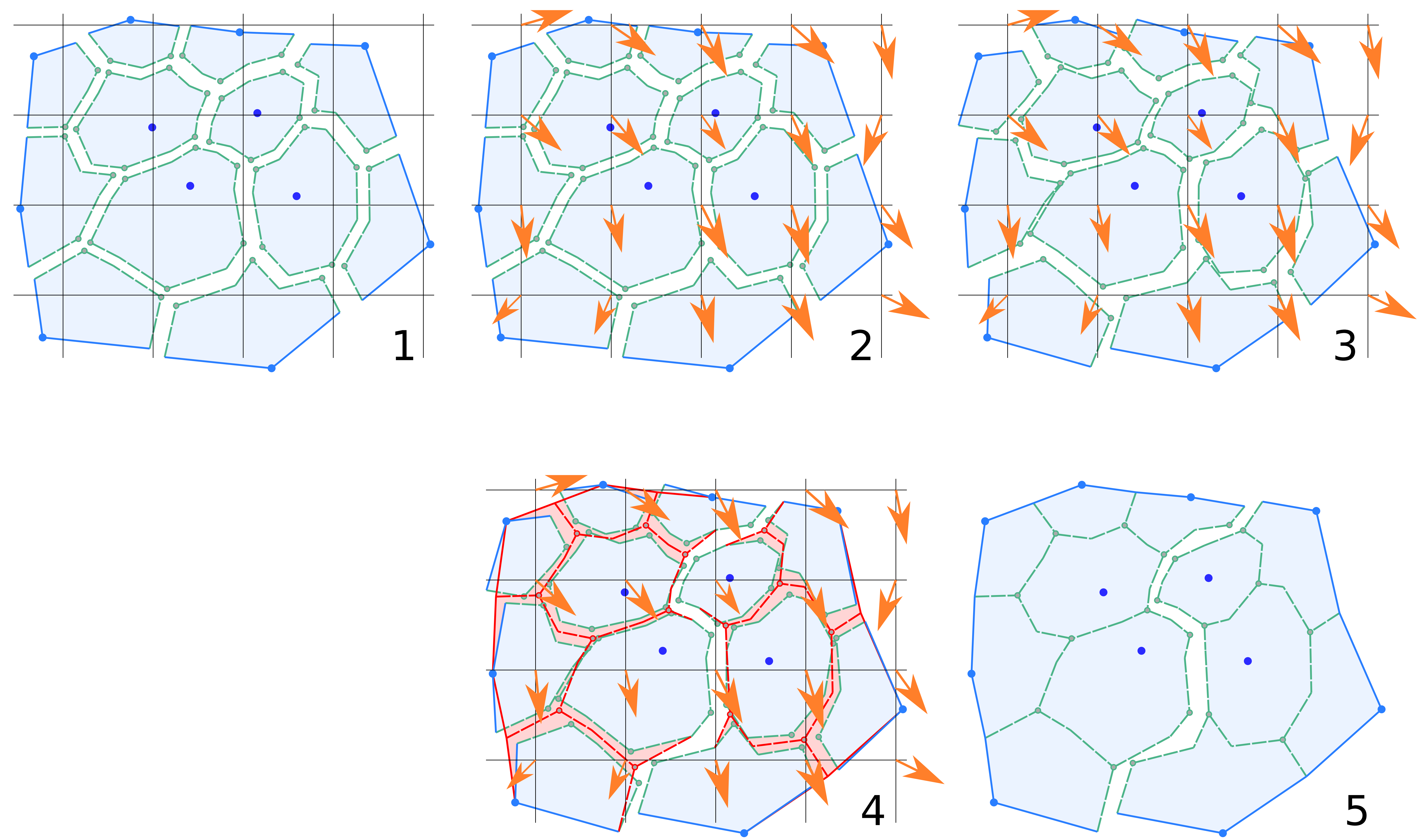


EXTRAPOLATING POSITIONS FOR ADDED VERTICES



EXTRAPOLATING POSITIONS FOR ADDED VERTICES

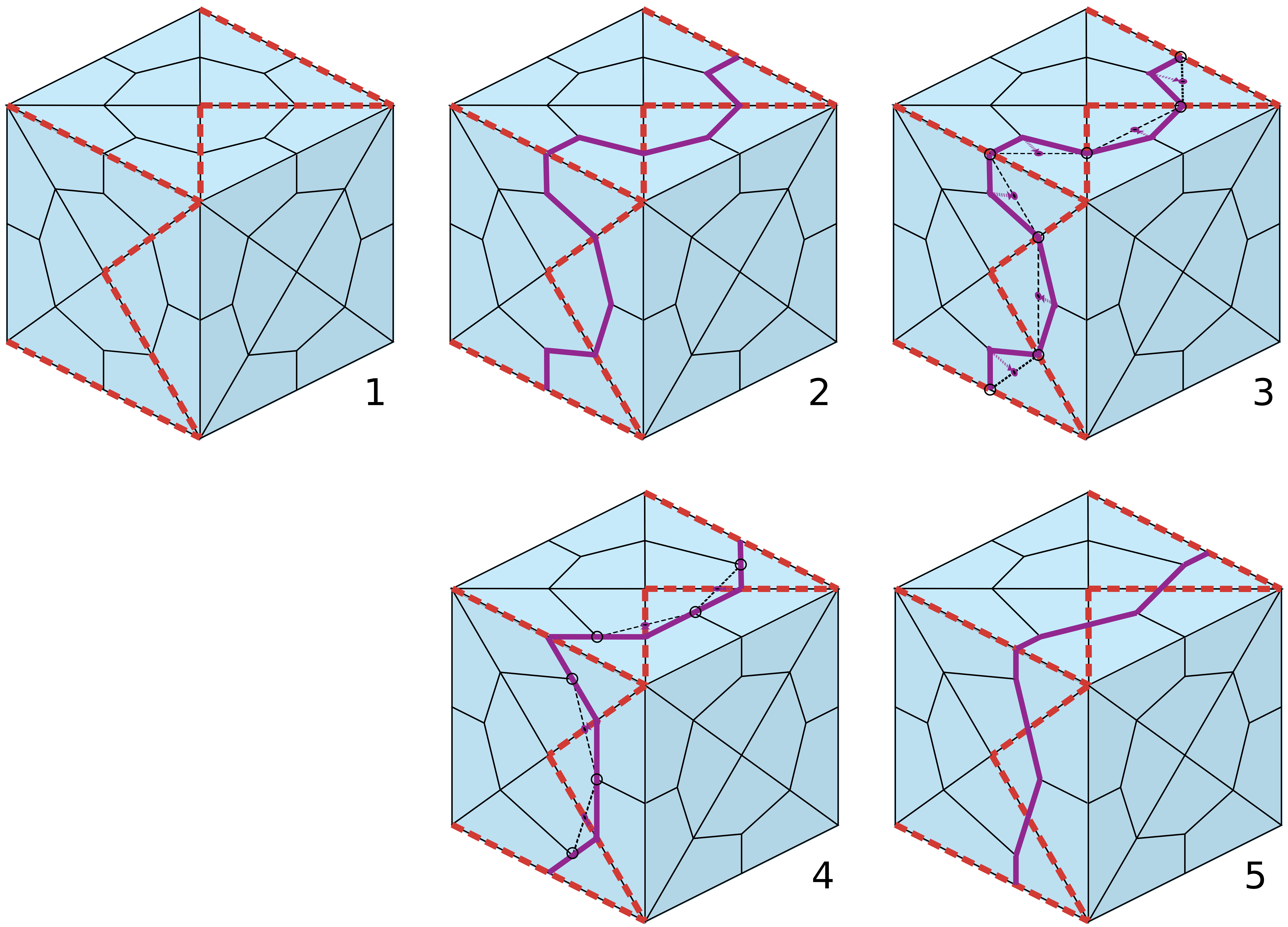
- Granular view
- Locally rigid motion
- Merging vertices based on topology

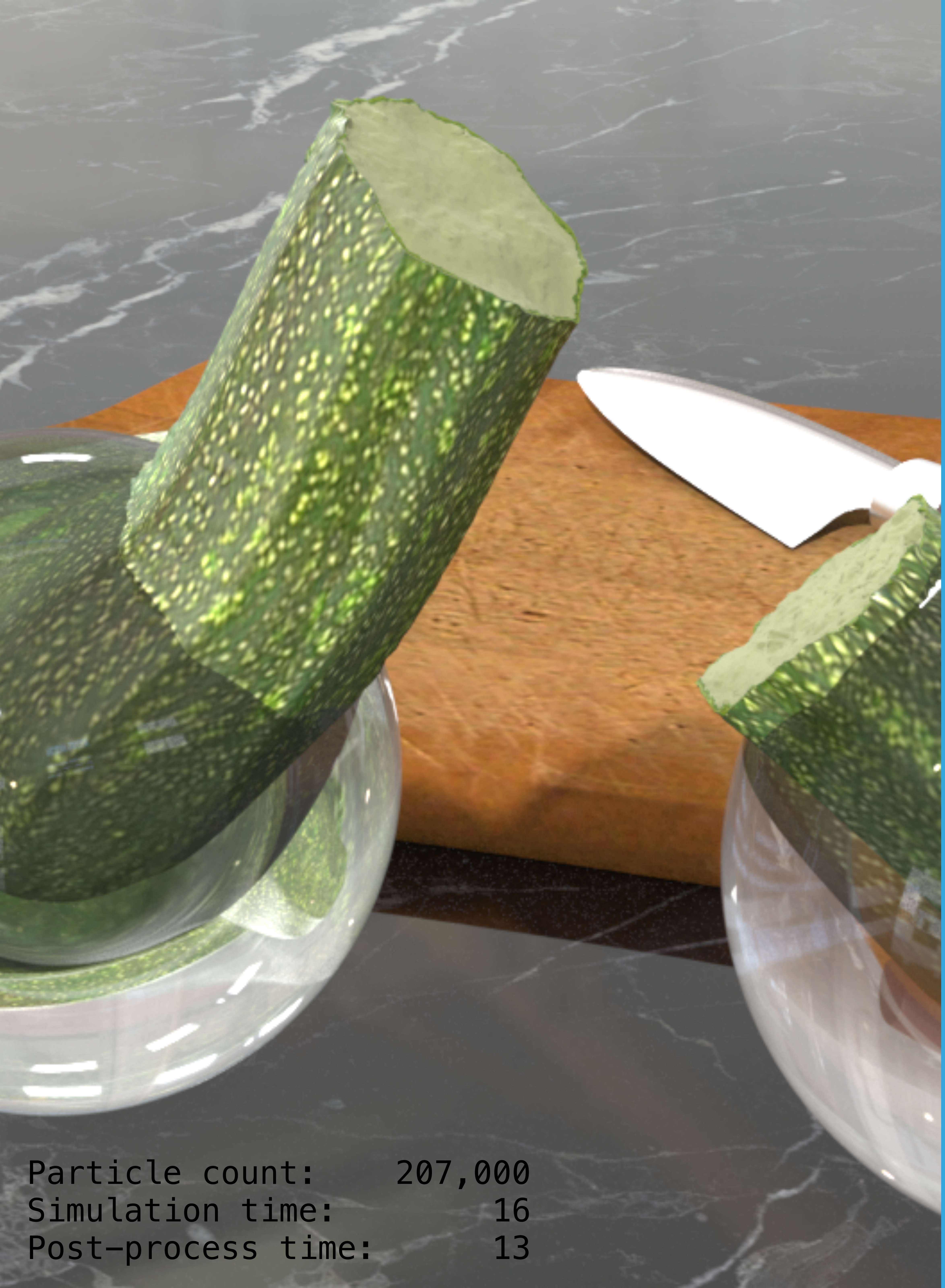


SMOOTHING CRACK SURFACE

SMOOTHING CRACK SURFACE

- Collect all ever broken edges
- Gauss-Siedel smoothing
- Smooth only the undeformed configuration




Particle count: 207,000
Simulation time: 16
Post-process time: 13

DISCUSSION

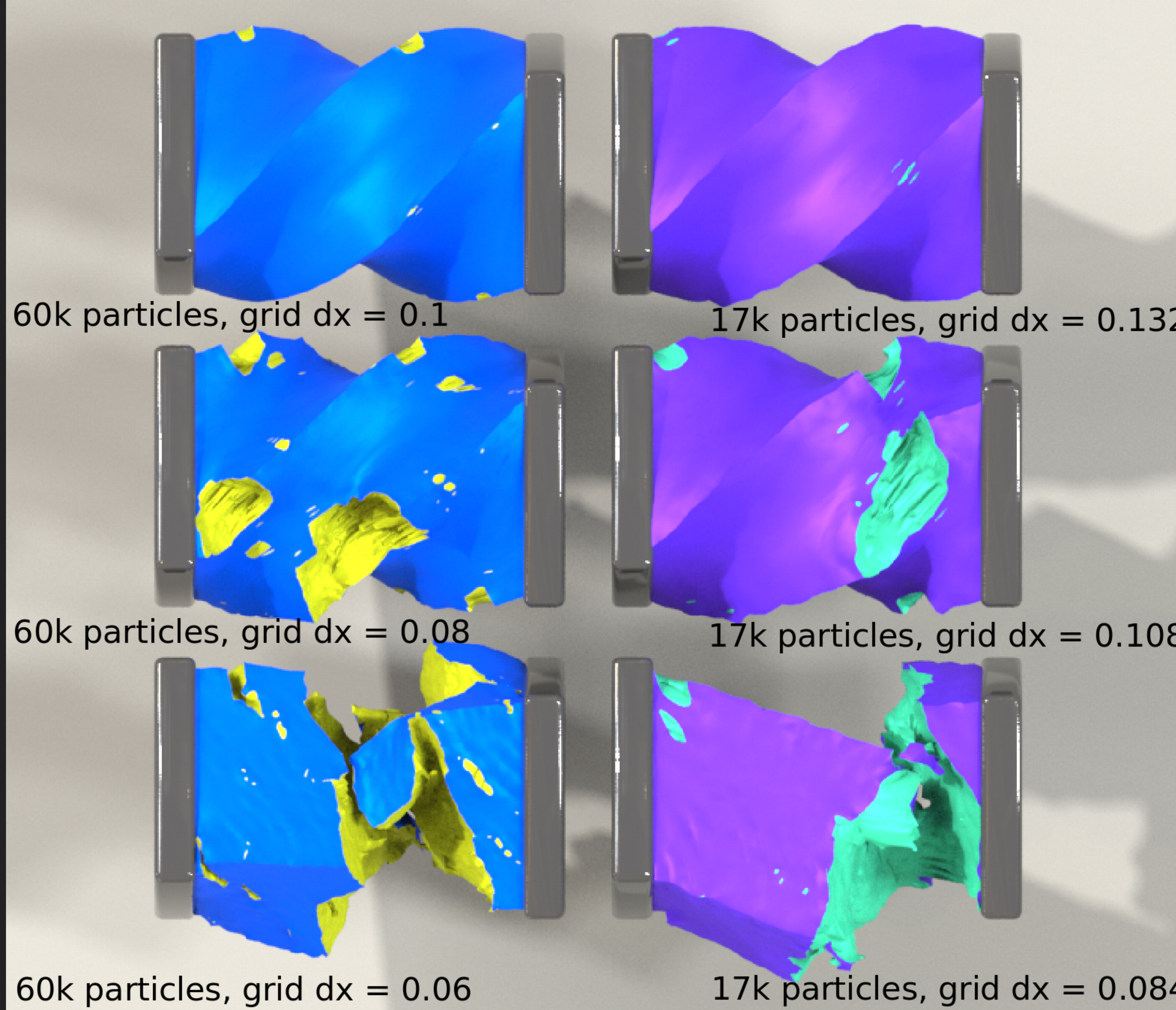
LIMITATIONS AND FUTURE DIRECTIONS

- Crack patterns can be affected by particle sampling density, mesh topology, grid resolution
- Finding appropriate parameters for edge-stretching threshold and crack smoothing iterations
- Exploring different yield surfaces and flow rules

MESH V.S. PARTICLE

Particle-based forces (grid velocity updated F)	Mesh-based forces (mesh geometry updated F)
Delaunay mesh for visualization	requires quality mesh for simulation
has artificial fracture	no artificial fracture
6-8 particles per cell	2 particles per cell
automatic self-collision	
easy coupling with other MPM material	


Small grid dx
Large grid dx
Lagrangian
Particle count: 8,000
Simulation time: 0.6
Post-process time: 0.5


60k particles, grid dx = 0.1
17k particles, grid dx = 0.132
60k particles, grid dx = 0.08
17k particles, grid dx = 0.108
60k particles, grid dx = 0.06
17k particles, grid dx = 0.084


Particle count: 933,000
Simulation time: 50
Post-process time: 5


Particle count: 50,000
Simulation time: 2
Post-process time: 3

ACKNOWLEDGEMENT

- The work is supported by NSF CCF-1422795, ONR (N000141110719, N000141210834), DOD (W81XWH15-1-0147), Intel STC-Visual Computing Grant (20112360) as well as a gift from Adobe Inc.