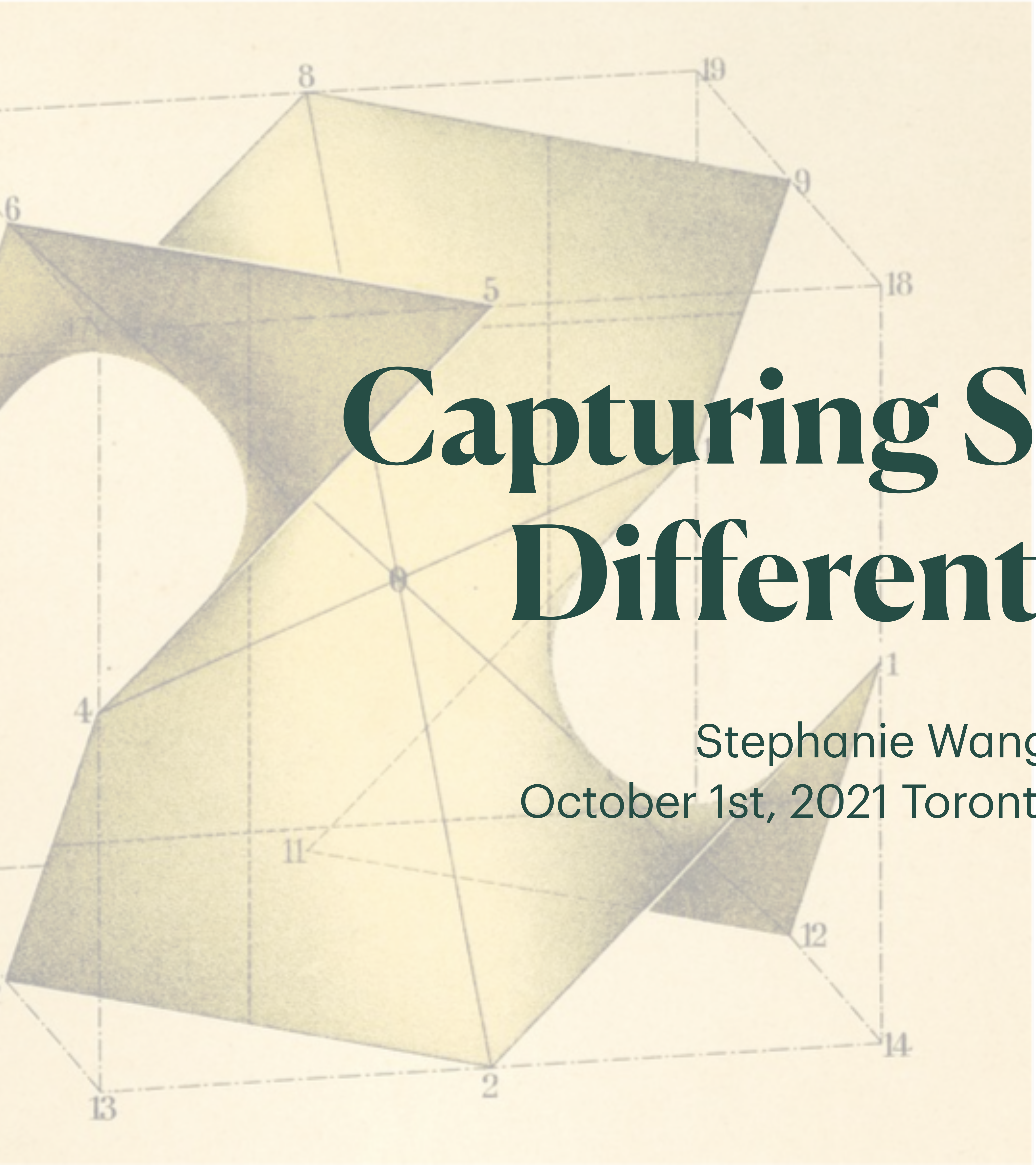

Capturing Surfaces with Differential Forms

Stephanie Wang, UC San Diego
October 1st, 2021 Toronto Geometry Colloquium

UC San Diego


UCLA

About me...

- Ph.D. in Mathematics, UCLA, 2020
- Dissertation: A Material Point Method for Elastoplasticity with Ductile Fracture and Frictional Contact
- Material Point Method (MPM) = multi-spices simulation with automatic collision

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- Postdoc at Computer Science and Engineering, UC San Diego
- Recent paper: Computing Minimal Surfaces with Differential Forms, Stephanie Wang and Albert Chern, ACM Transactions on Graphics (TOG)—Proceedings of ACM SIGGRAPH 2021.

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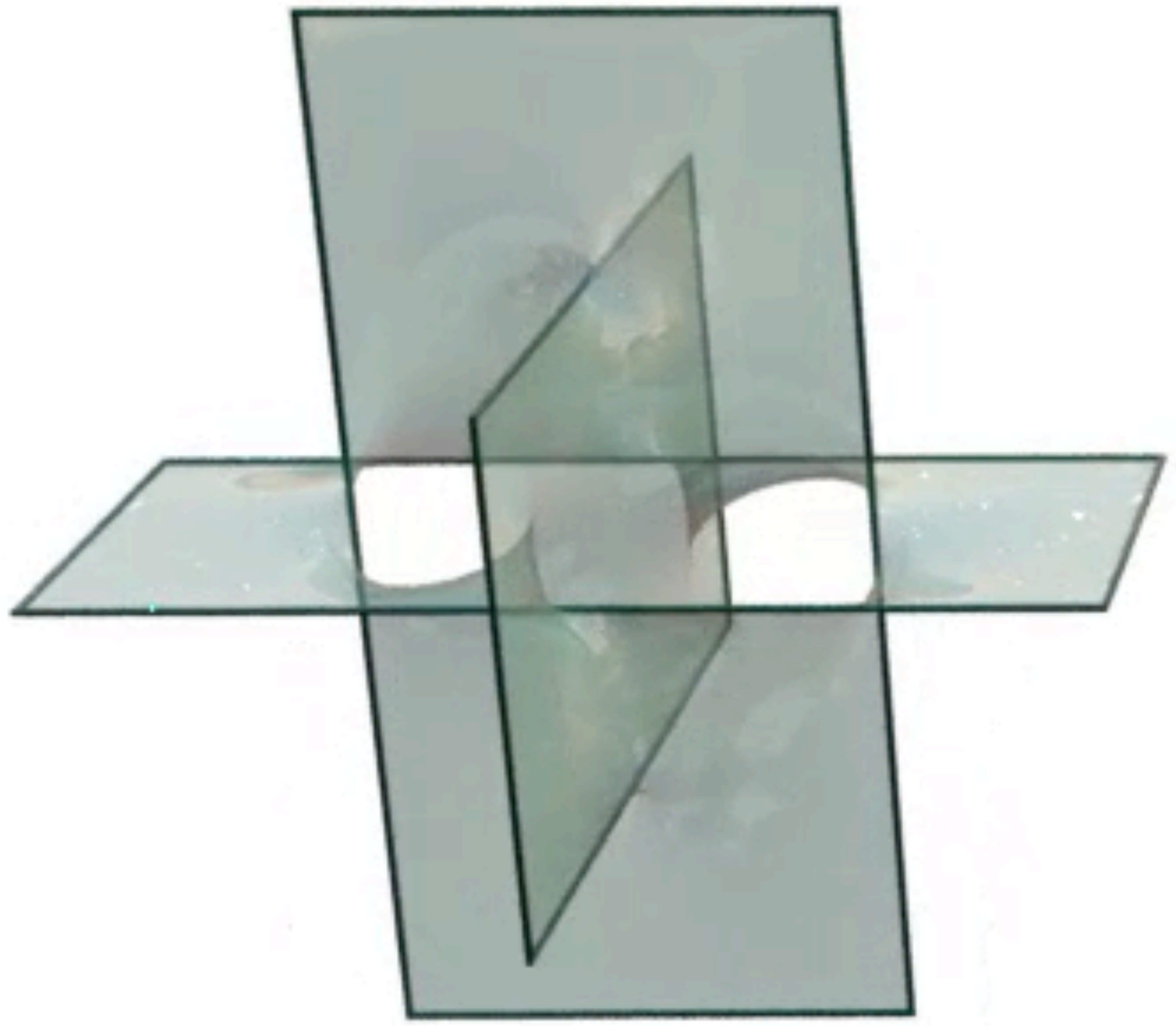

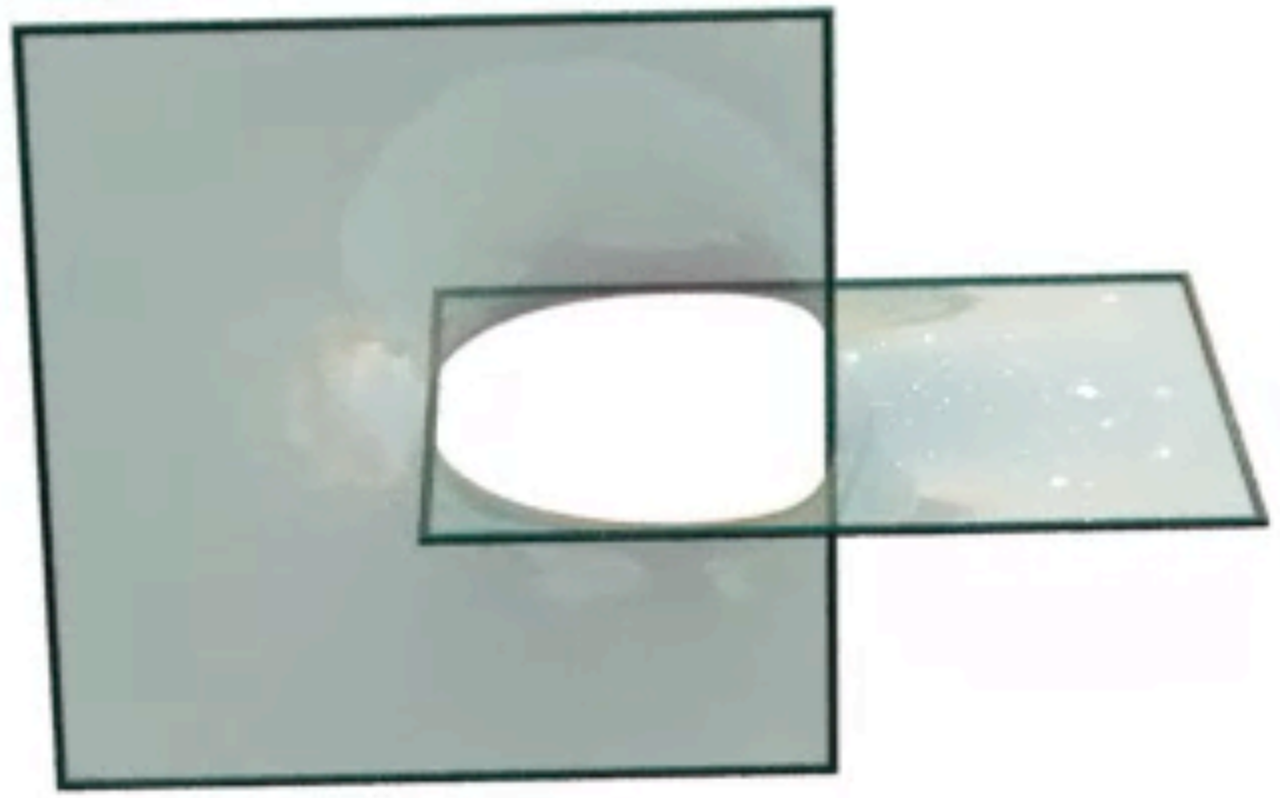

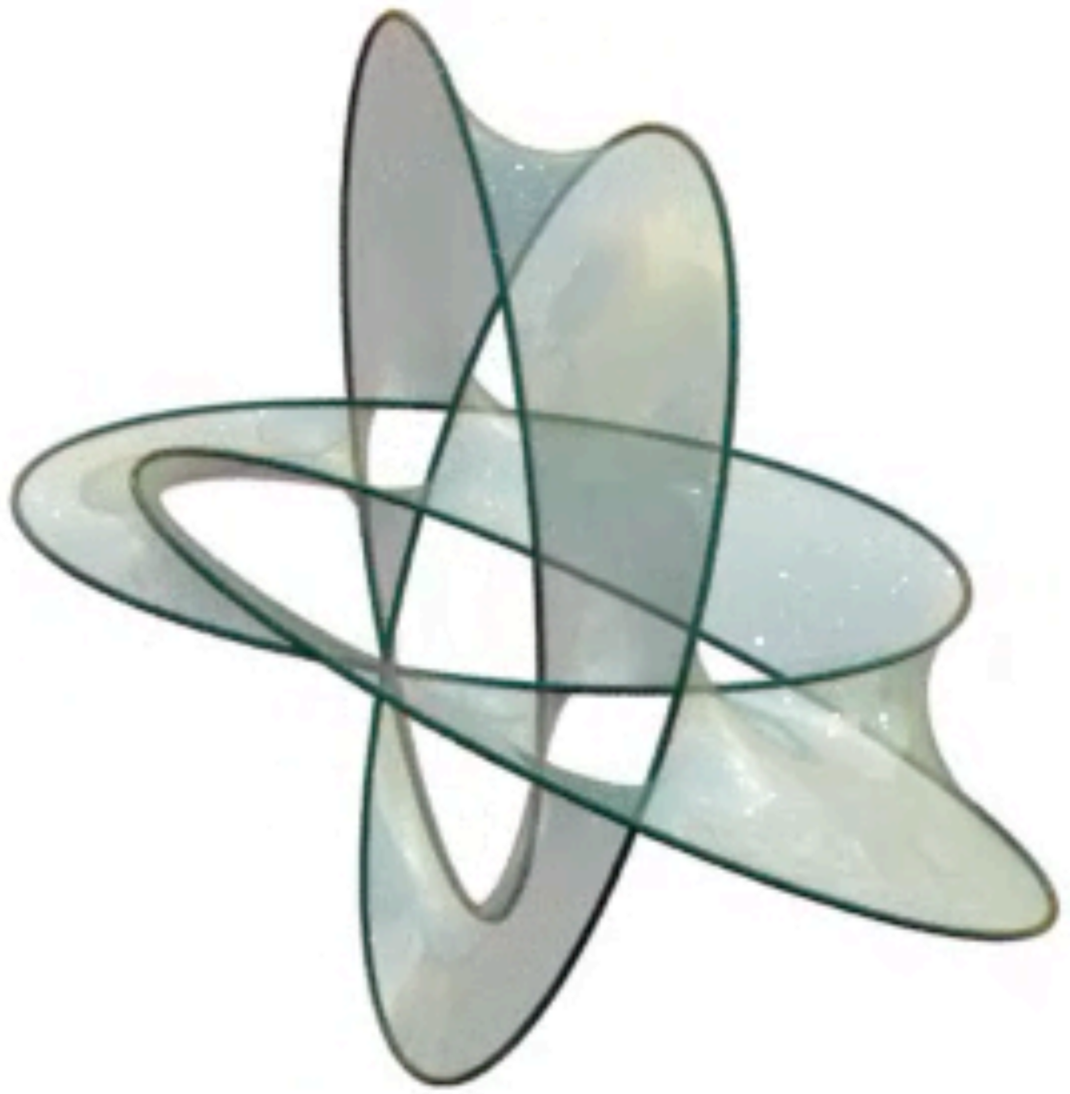

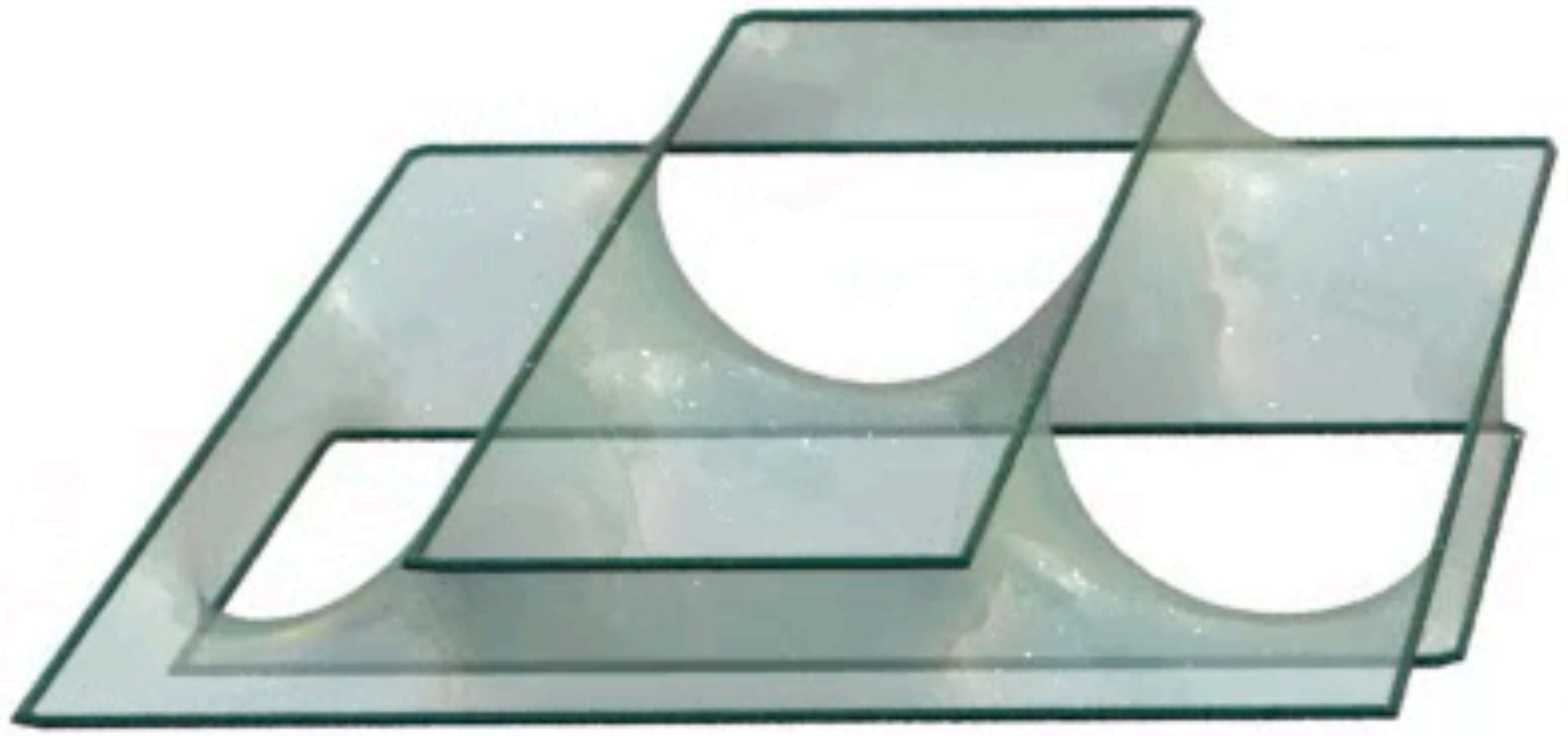

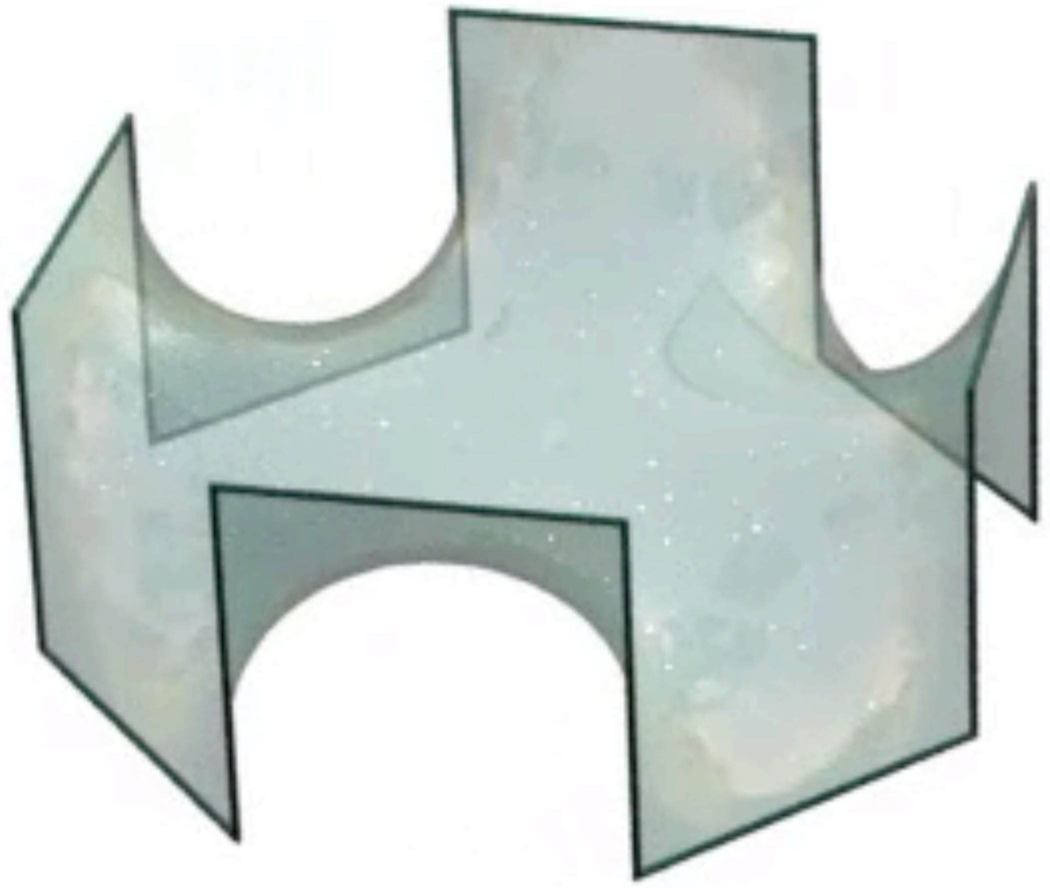

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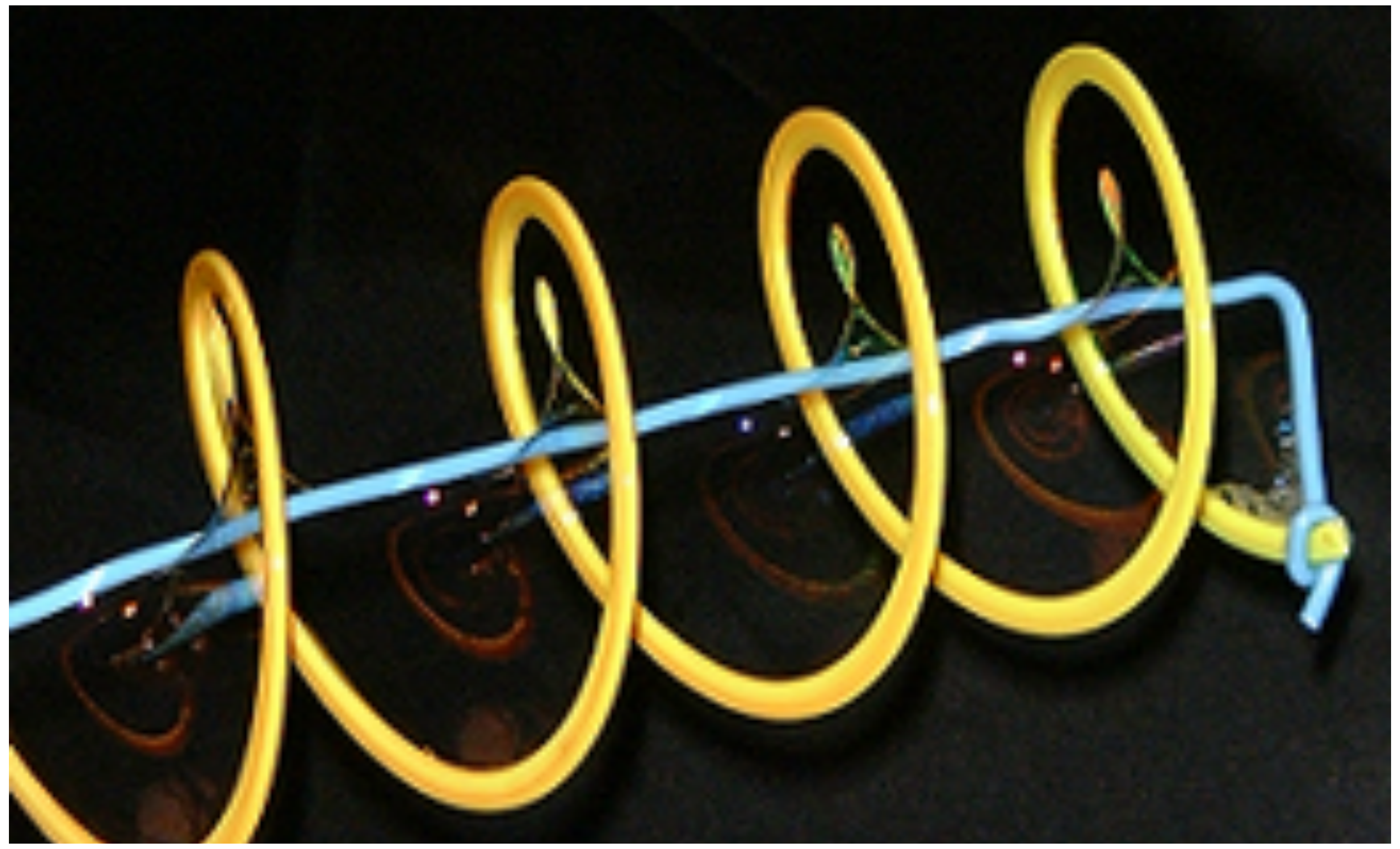
Helicoid formed by soap film on a helical frame PC: Blinking Spirit



RUBATO by Eva Hild PC: David Eppstein

Münich Olympiapark PC: Tiia Monto

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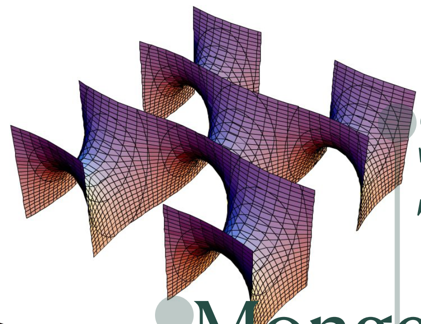
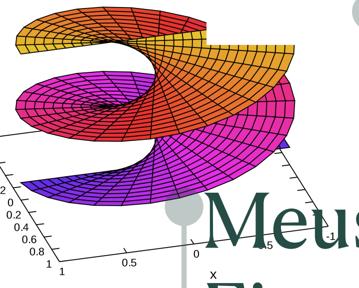
A brief history of minimal surfaces

The chronicle



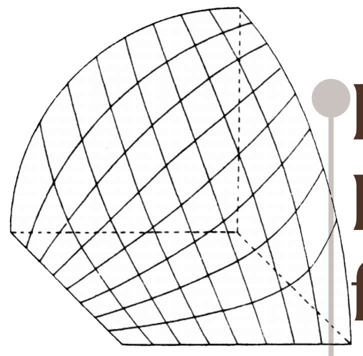
Scherk 1834
Third example of minimal surfaces (periodic)



Monge and Legendre 1795
Representation formulas for minimal surfaces

Meusnier 1776
First non-trivial example: helicoid and catenoid

Lagrange 1760 $k + \left(\frac{d\mathrm{P}}{dx}\right) + \left(\frac{d\mathrm{Q}}{dy}\right) = 0,$
Euler-Lagrange equation of surface area functional for height field



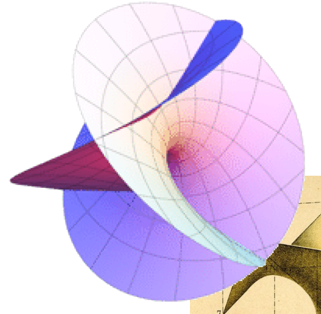
Douglas 1927
Finite difference with multigrid acceleration
for height field with arbitrary boundary value

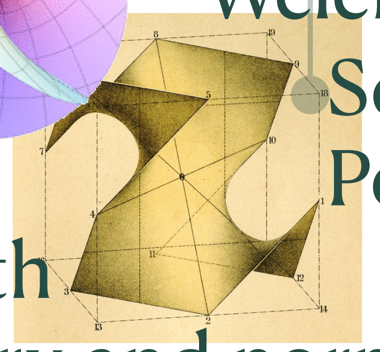
Bernstein 1914
Boundaryless function graphs in R^3 are planes

Wilson 1961
Finite difference with conformal map

Concus 1967
Finite difference for height f

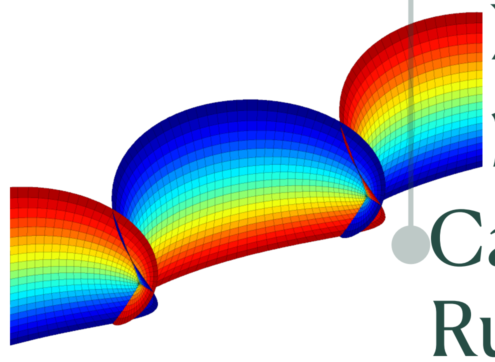
Hinata, Shimasaki, and Ki
Finite element for height f





Enneper and Weierstrass 1863?
Weierstrass representation and Enneper surface

Schwarz 1865-1867
Periodic Schwarz surface families



Björling 1844
Minimal surface with
prescribed boundary and normal

Catalan 1842-43
Ruled minimal surface (helicoid)

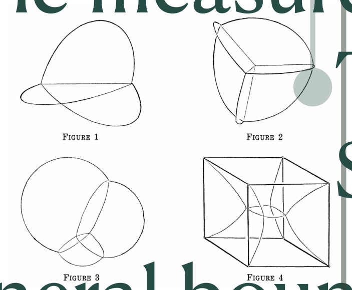
Plateau 1873
Plateau's laws of soap films
(including nonmanifold soap films)

Kelvin 1887
Bubble foam packing problem

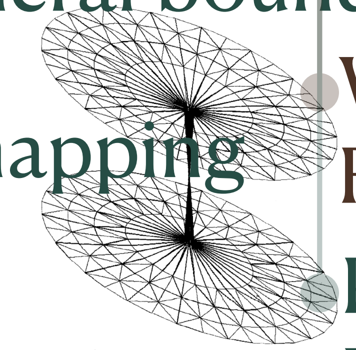
Douglas 1931, Rado 1931 (independently)
Existence for disks based on conformal mapping

Courant 1936-40
Existence for more general boundary curves with con

Federer and Fleming 1960
Geometric measure theory

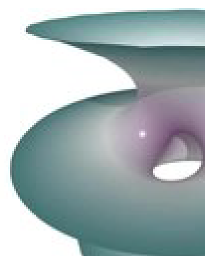


Taylor 1976
Soap film singularities



Wagner 1977
Finite element for ge

Parks 1977 (1986 boo
Levelset of functions
with boundary on bo



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Plateau problem

finding the minimal surface with a given boundary curve

- M: a (3-dimensional) ambient manifold.
- Given a closed boundary curve $\Gamma \hookrightarrow M$ (that is, $\partial\Gamma = \emptyset$).
- Find a surface $\Sigma \hookrightarrow M$ that

$$\begin{aligned} \text{minimize} \quad & \text{Area}(\Sigma) \\ \text{s.t.} \quad & \partial\Sigma = \Gamma \end{aligned}$$

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“Area functional is not convex.”

—well-known geometric processing fact.

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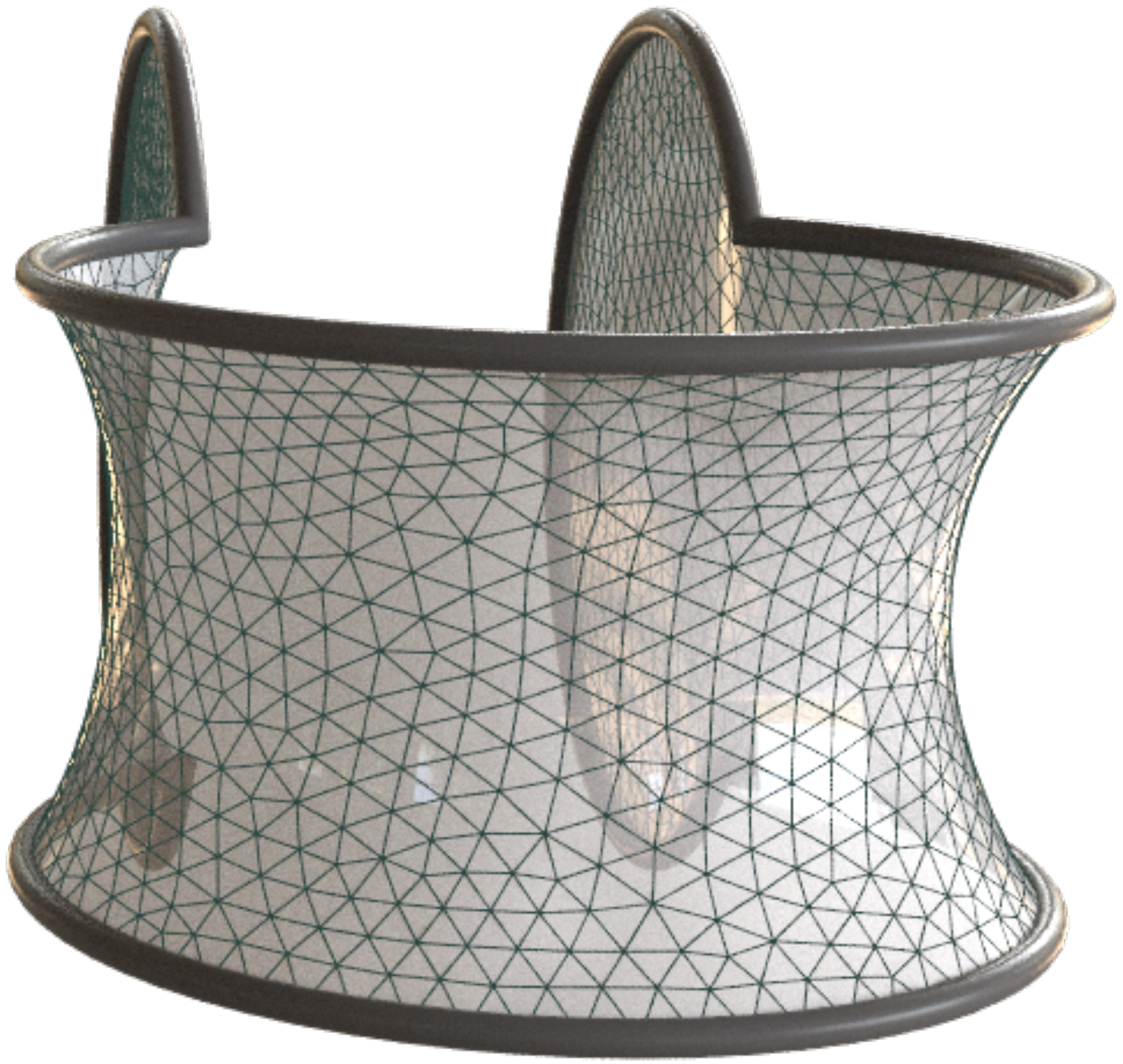

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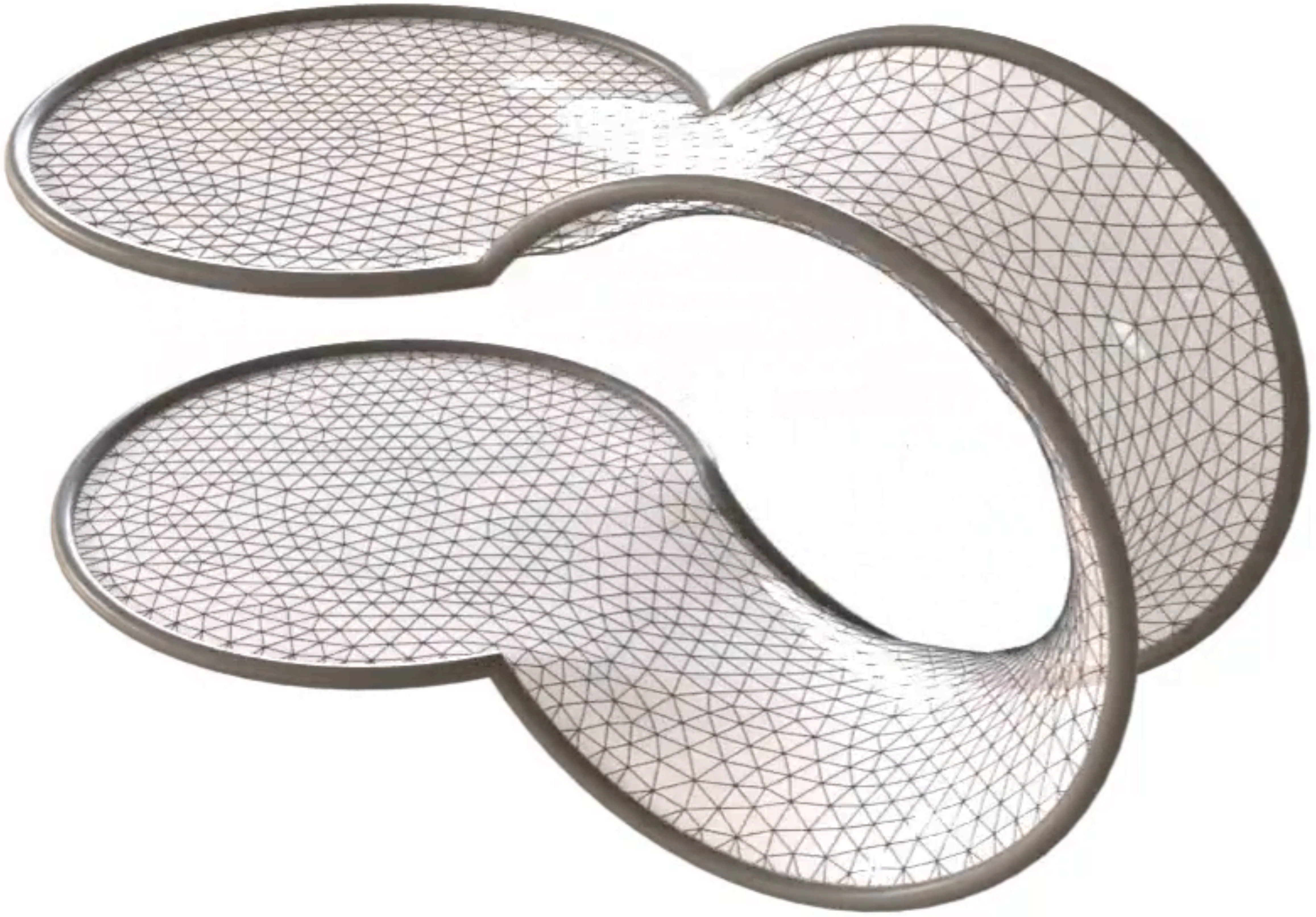

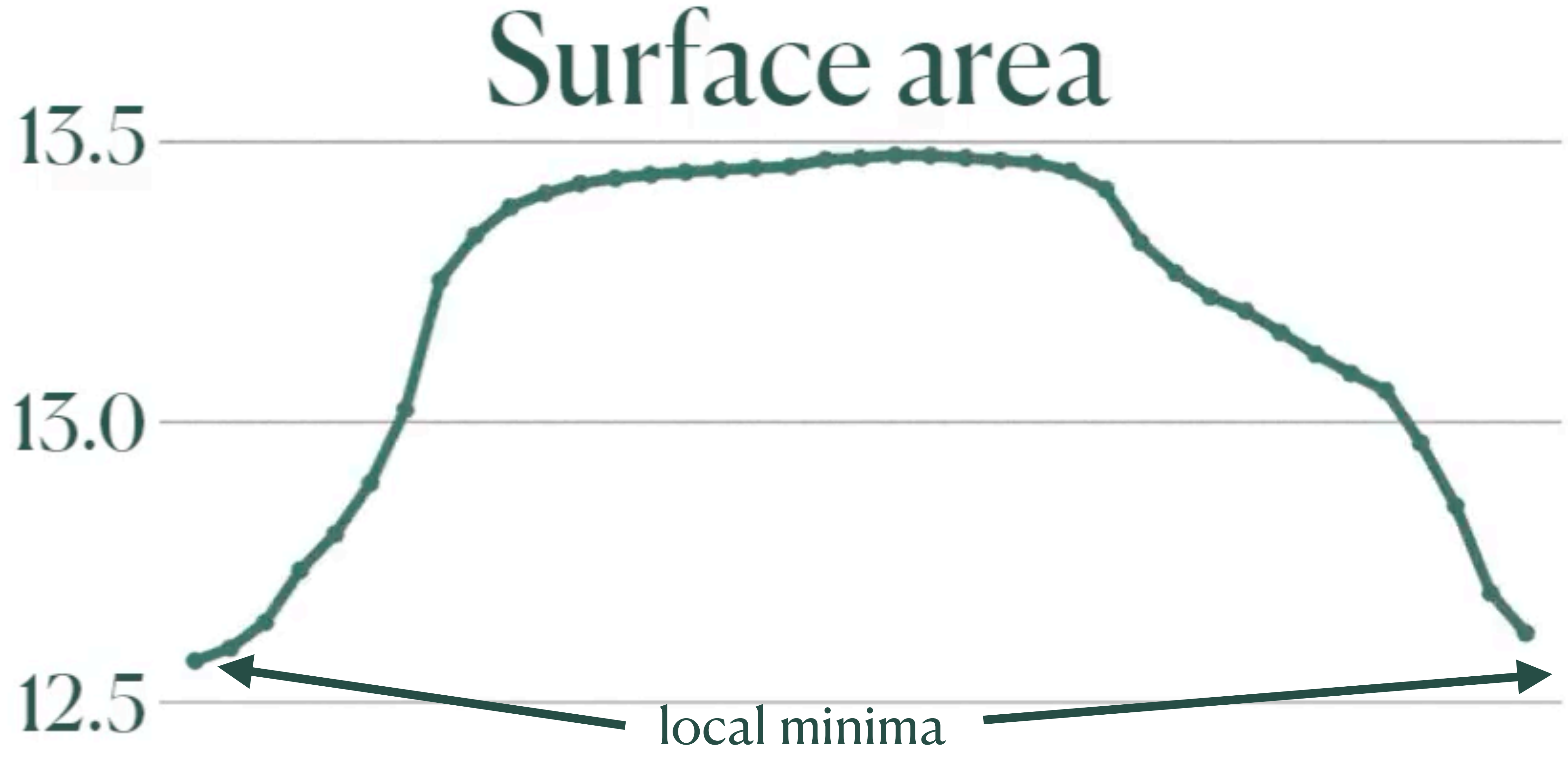

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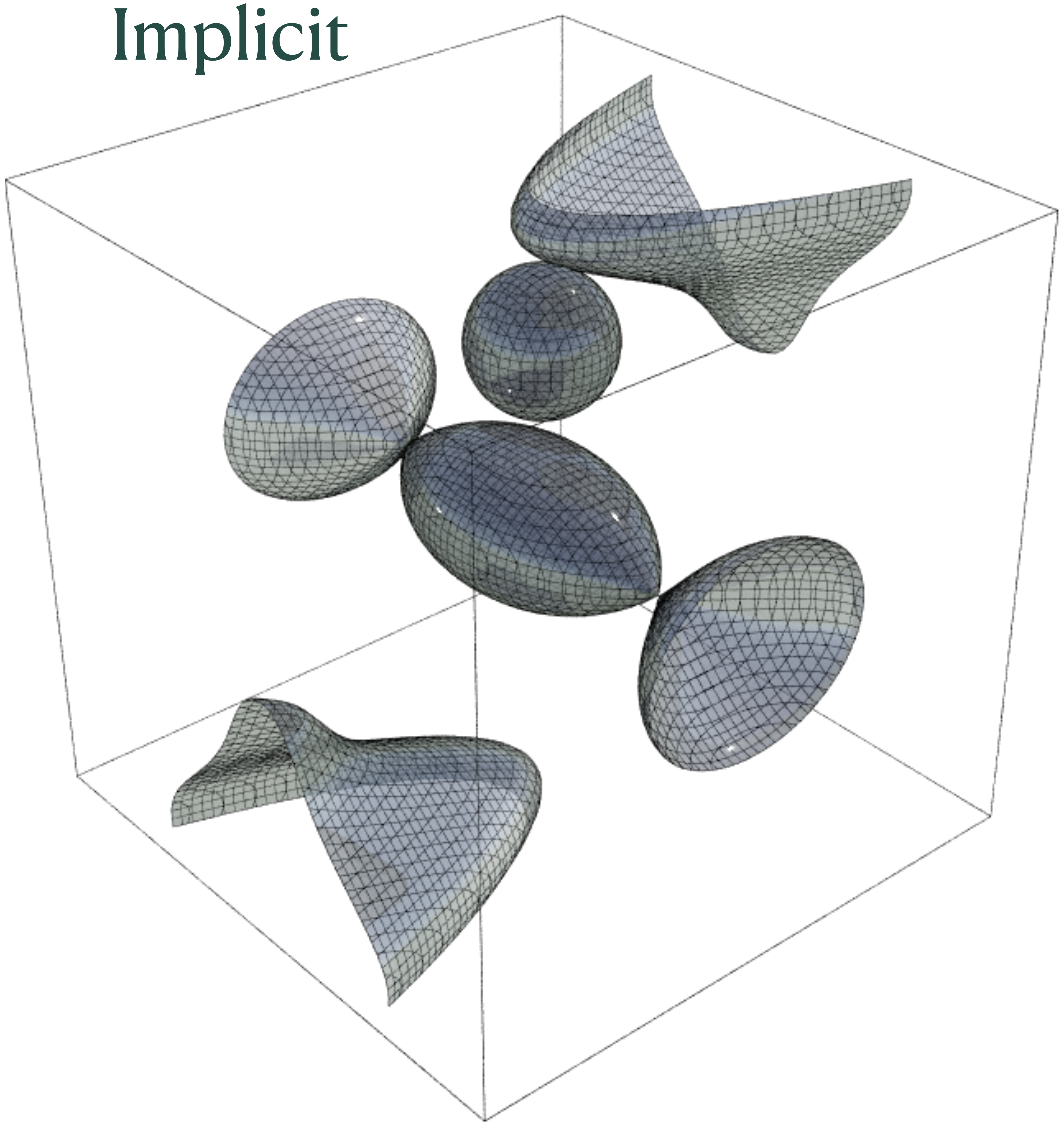
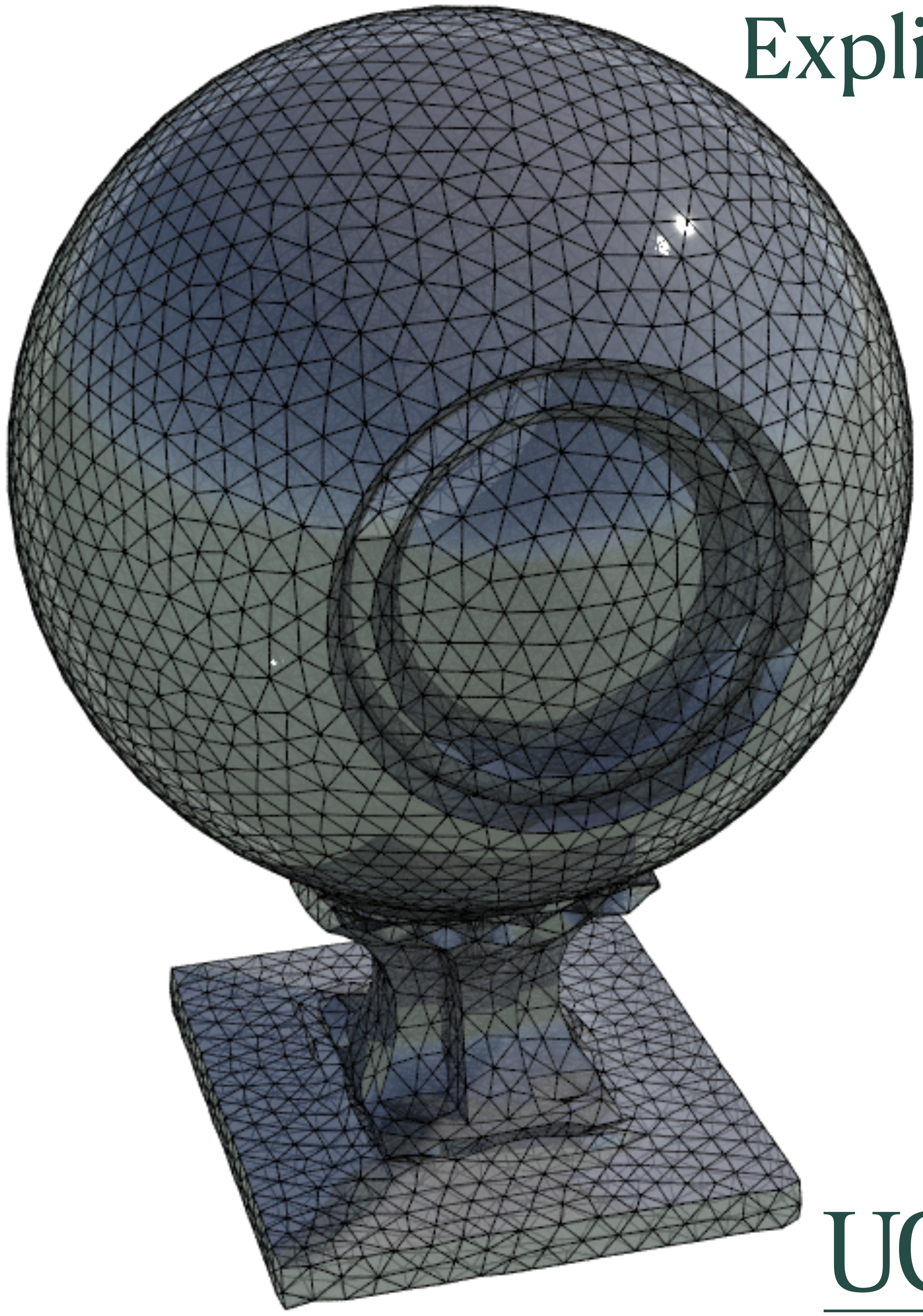
Surface representations

Implicit



Explicit



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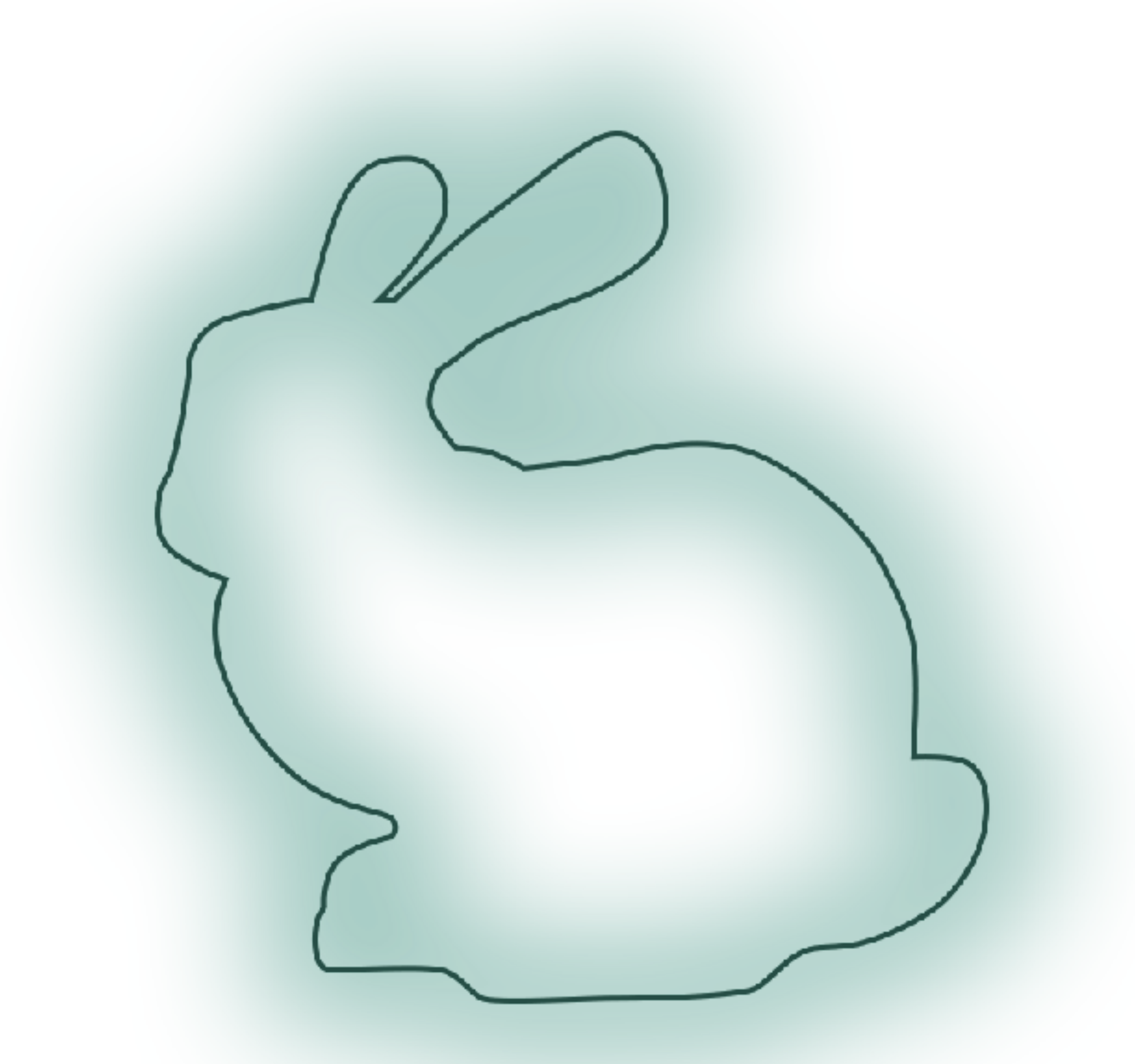
Level-set-based methods

no boundary for level sets

- Surface is implicitly stored with a spatial function $f : M \to \mathbb{R}$
  - Level set $\Sigma = \{\mathbf{x} \in M | f(\mathbf{x}) = 0\}$



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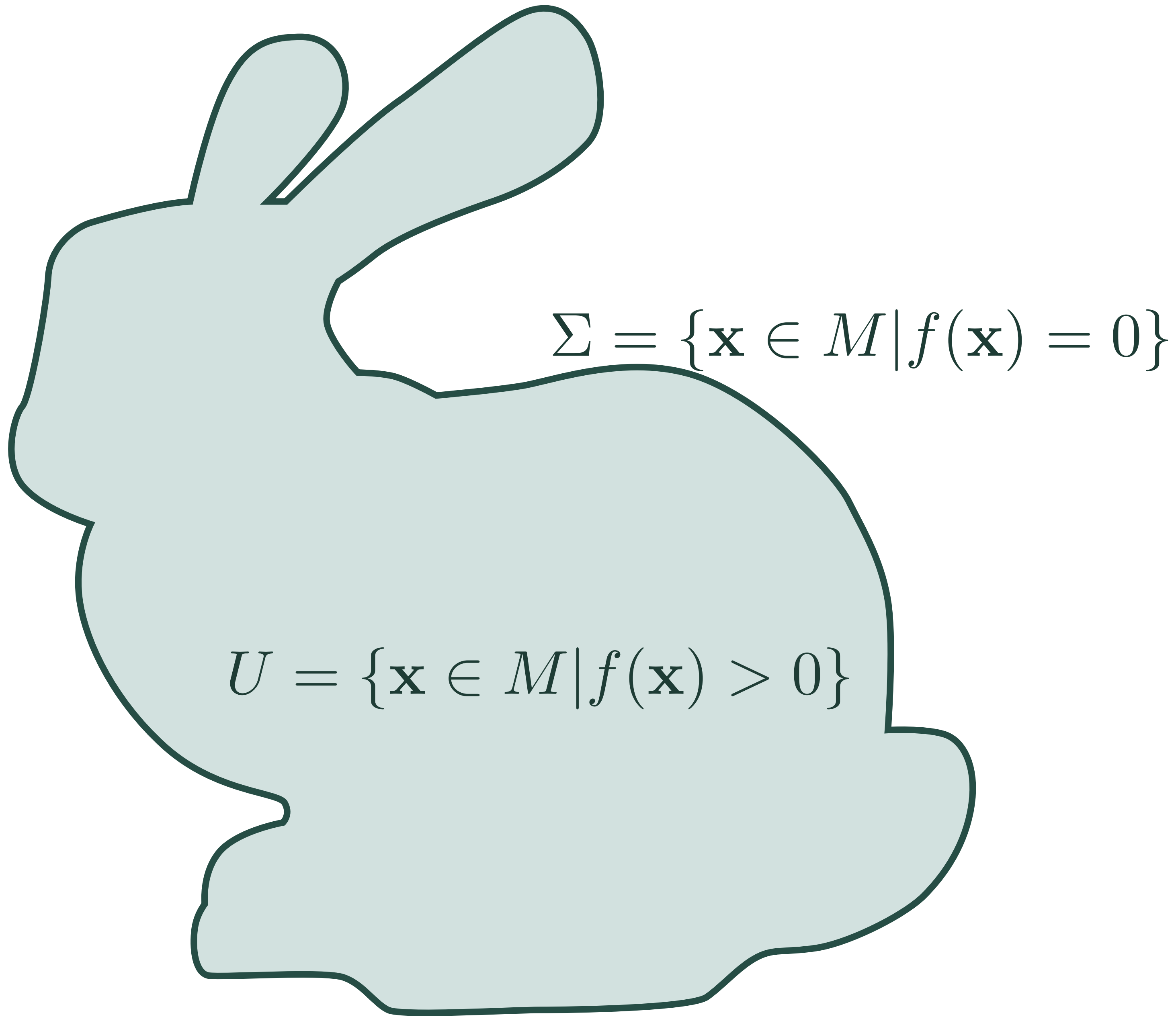
Level-set-based methods

no boundary for level sets

- Surface is implicitly stored with a spatial function $f : M \to \mathbb{R}$
  - Level set $\Sigma = \{\mathbf{x} \in M | f(\mathbf{x}) = 0\}$
- Inherently, $\Sigma = \partial\{\mathbf{x} \in M | f(\mathbf{x}) > 0\} = \partial U$ boundary of an open region.
  - $\partial\Sigma = \partial\partial U = \varnothing$.

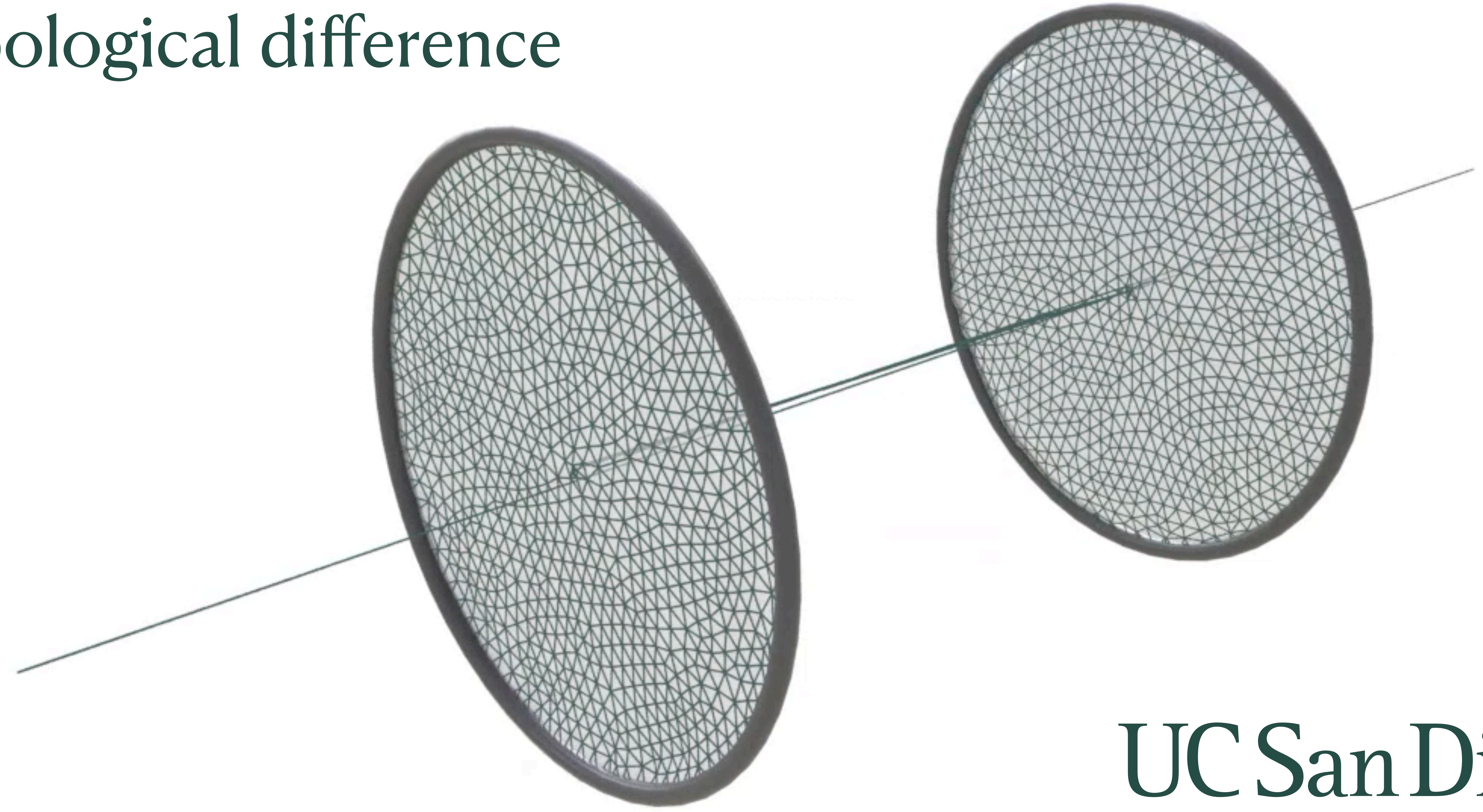


Mesh-based methods

- Discrete curvature flow
- Ill-conditioned Laplacian for low quality mesh

Mesh-based methods

- Discrete curvature flow
- Ill-conditioned Laplacian for low quality mesh
- Discrete operation to resolve topological difference



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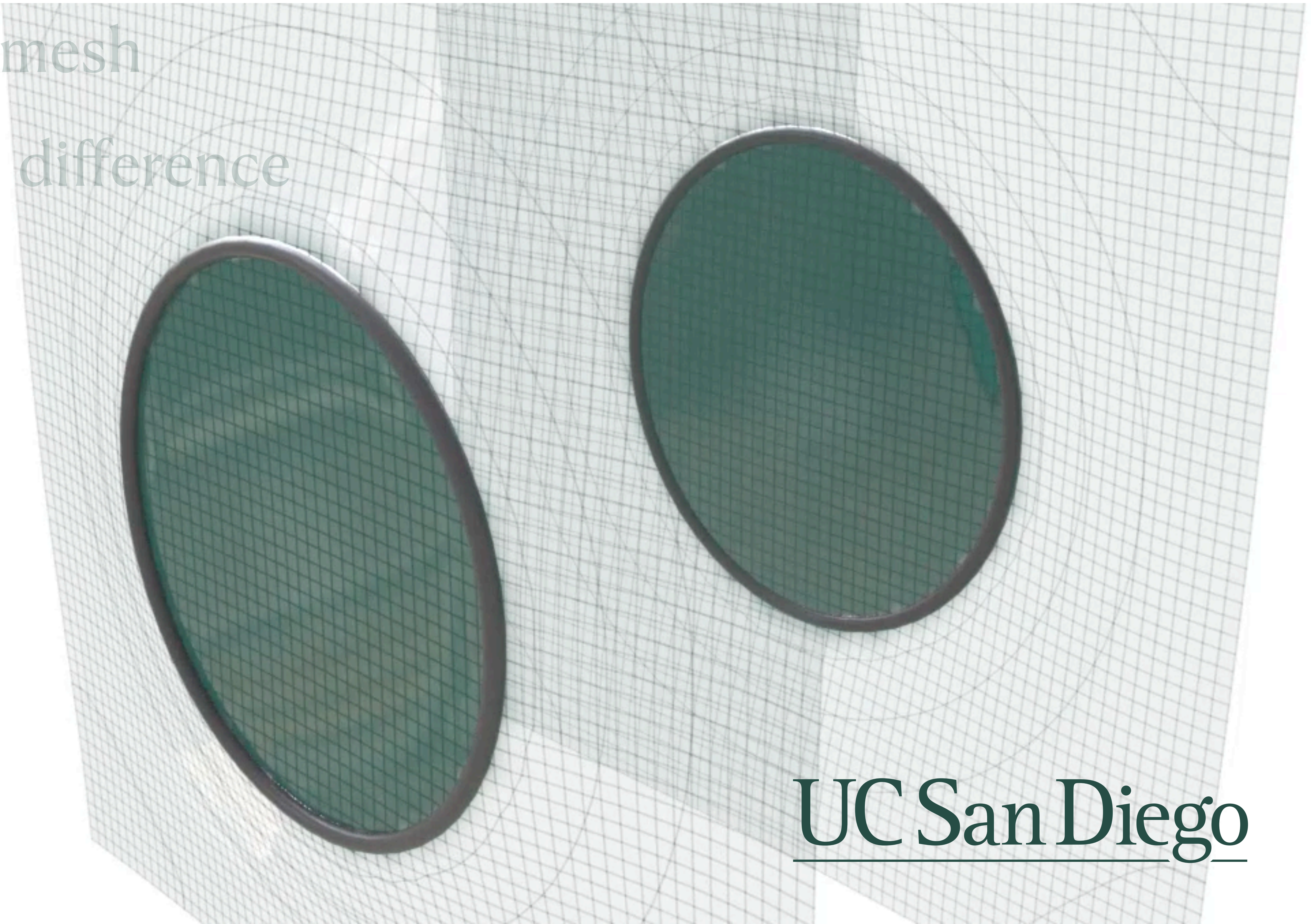
Our method

connectivity represented differently

- Discrete curvature flow
- Ill-conditioned Laplacian for low quality mesh
- Discrete operation to resolve topological difference



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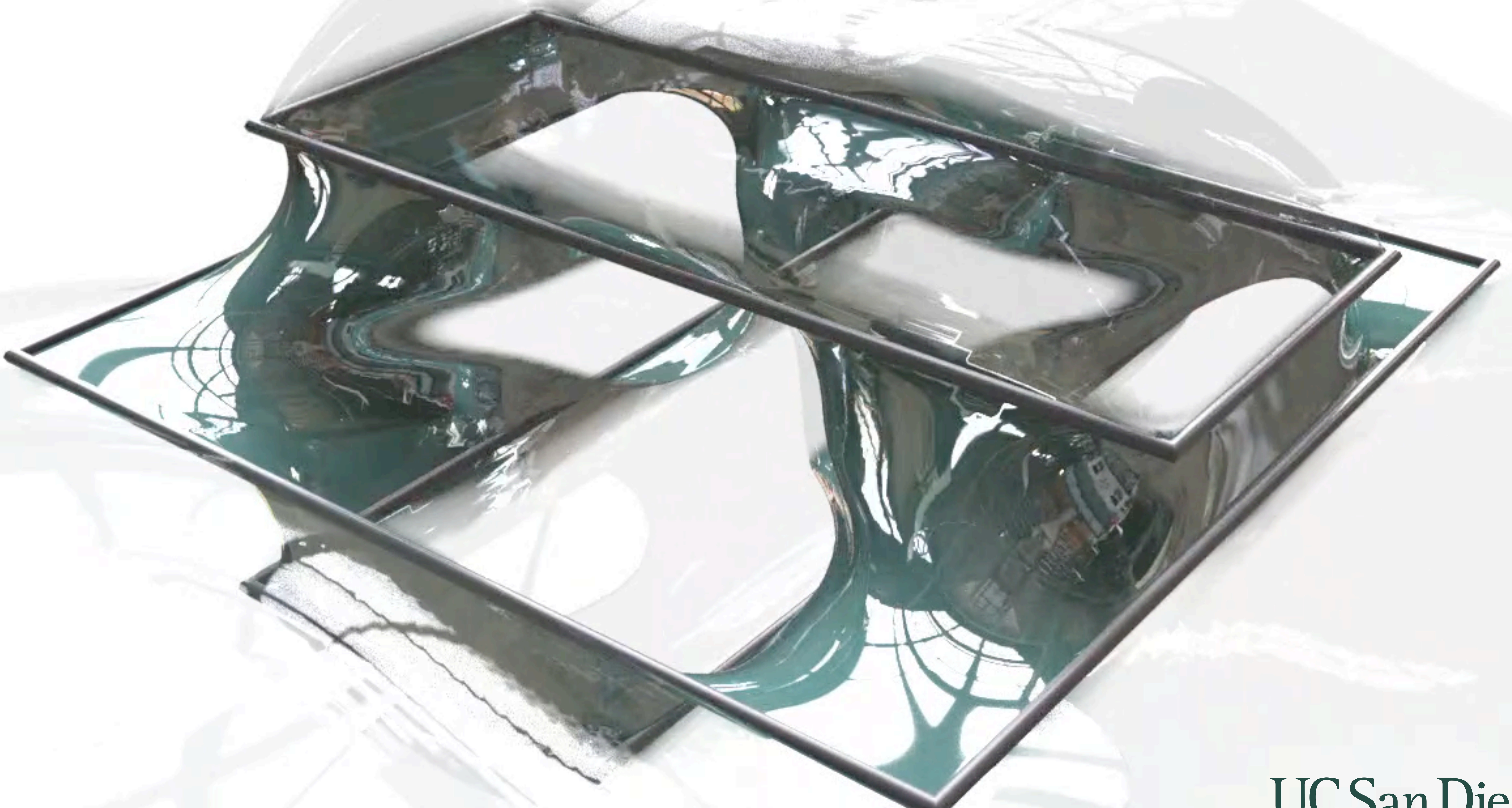
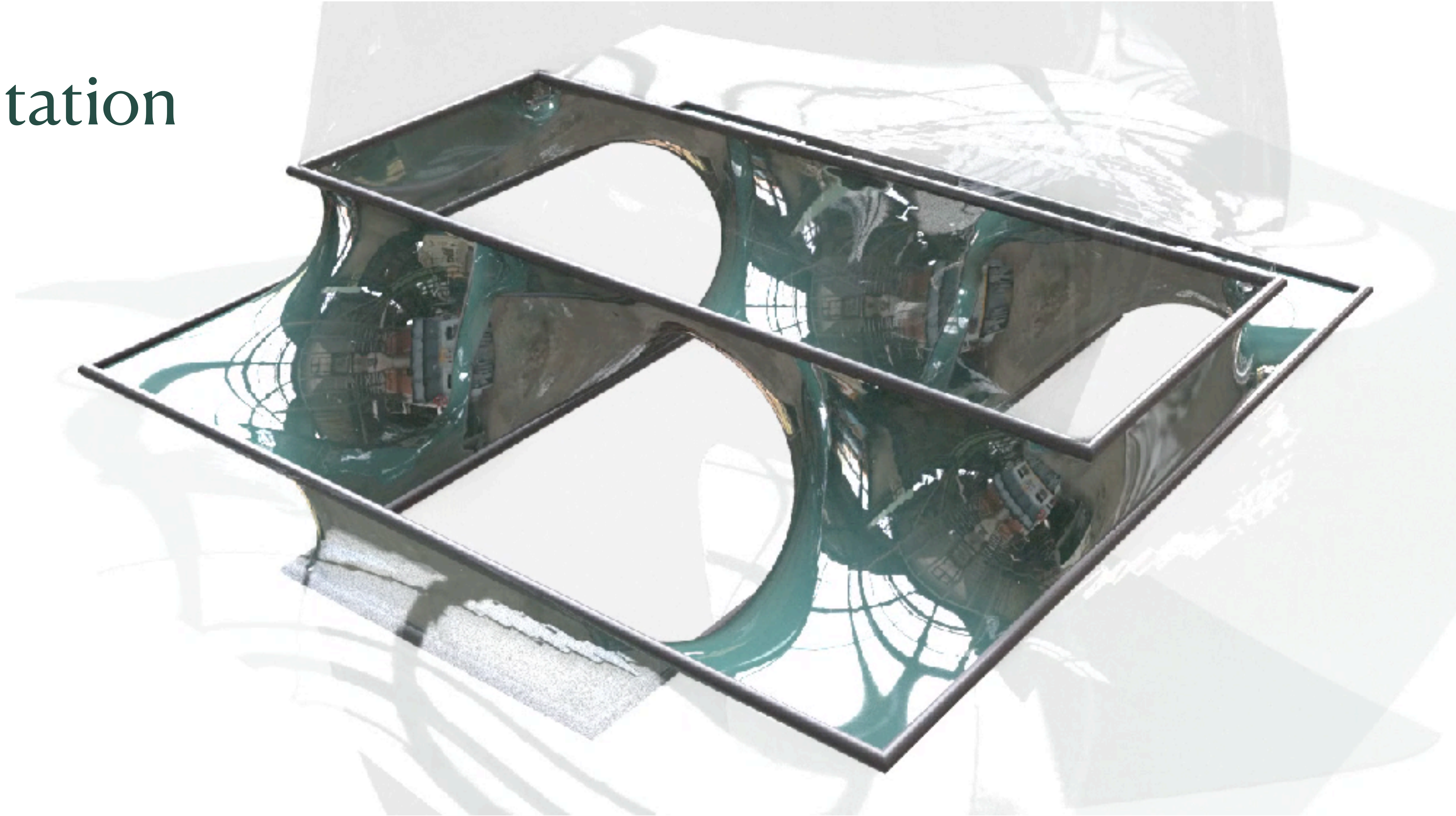


Outline

- Differential forms and surface representation
- Plateau problem
- Fast ADMM algorithms and results
- Generalized mass norm minimization



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Differential forms

$$\int_U f(x, y, z)\,dx\,dy\,dz$$ 3-form

(U: a volumetric region)

$$\int_\Sigma F\,dy\,dz + G\,dz\,dx + H\,dx\,dy$$ 2-form

(Σ: a surface)

$$\int_\Gamma f\,dx + g\,dy + h\,dz$$ 1-form

(Γ:a curve)

$$f(x_0, y_0, z_0) \quad f(x, y, z)$$ 0-form

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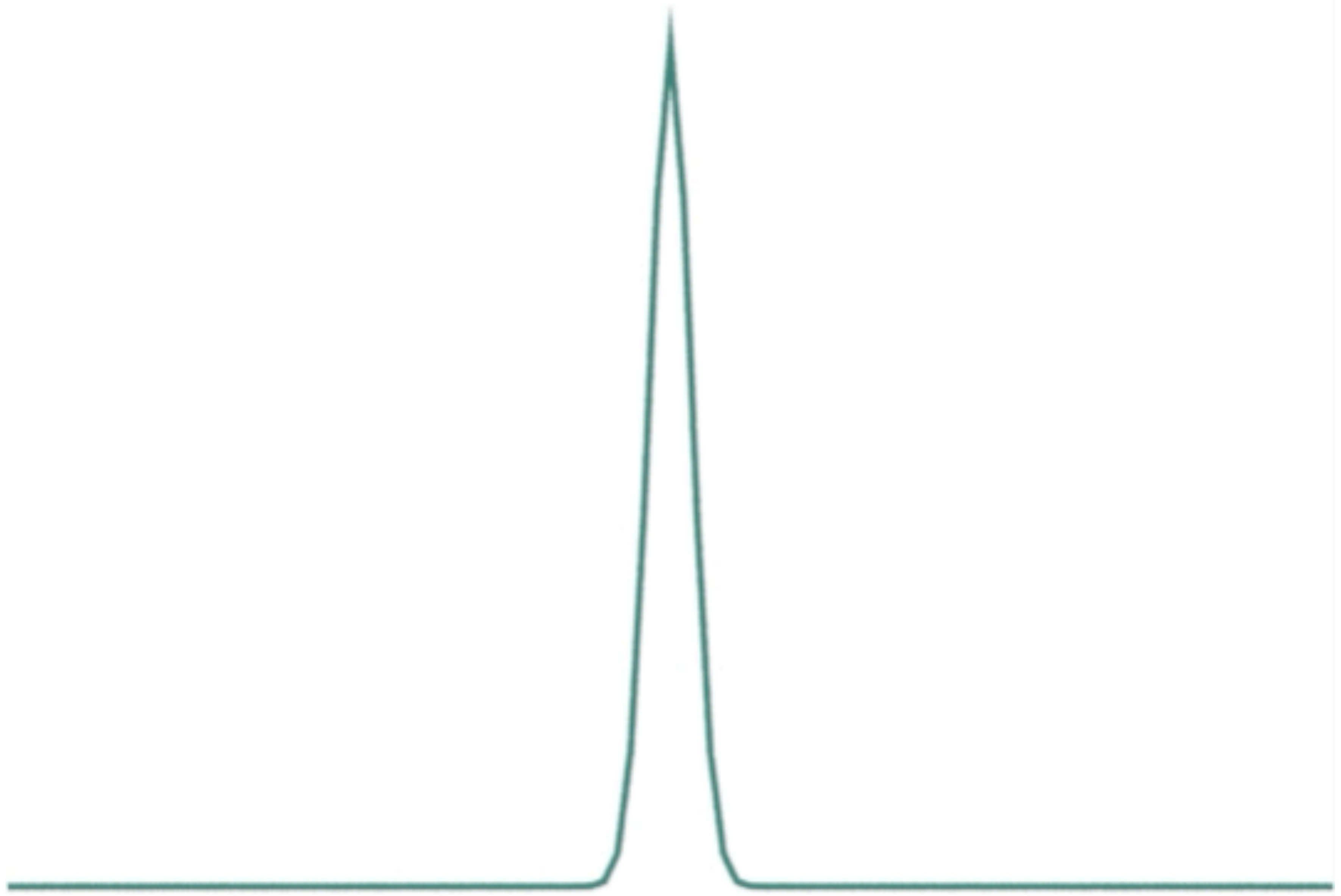
Differential forms and Dirac delta

$\delta_p(x, y, z)\,dxdydz$ 3-form



$$\int_M \varphi(x,y,z) f(x,y,z)\,dxdydz$$

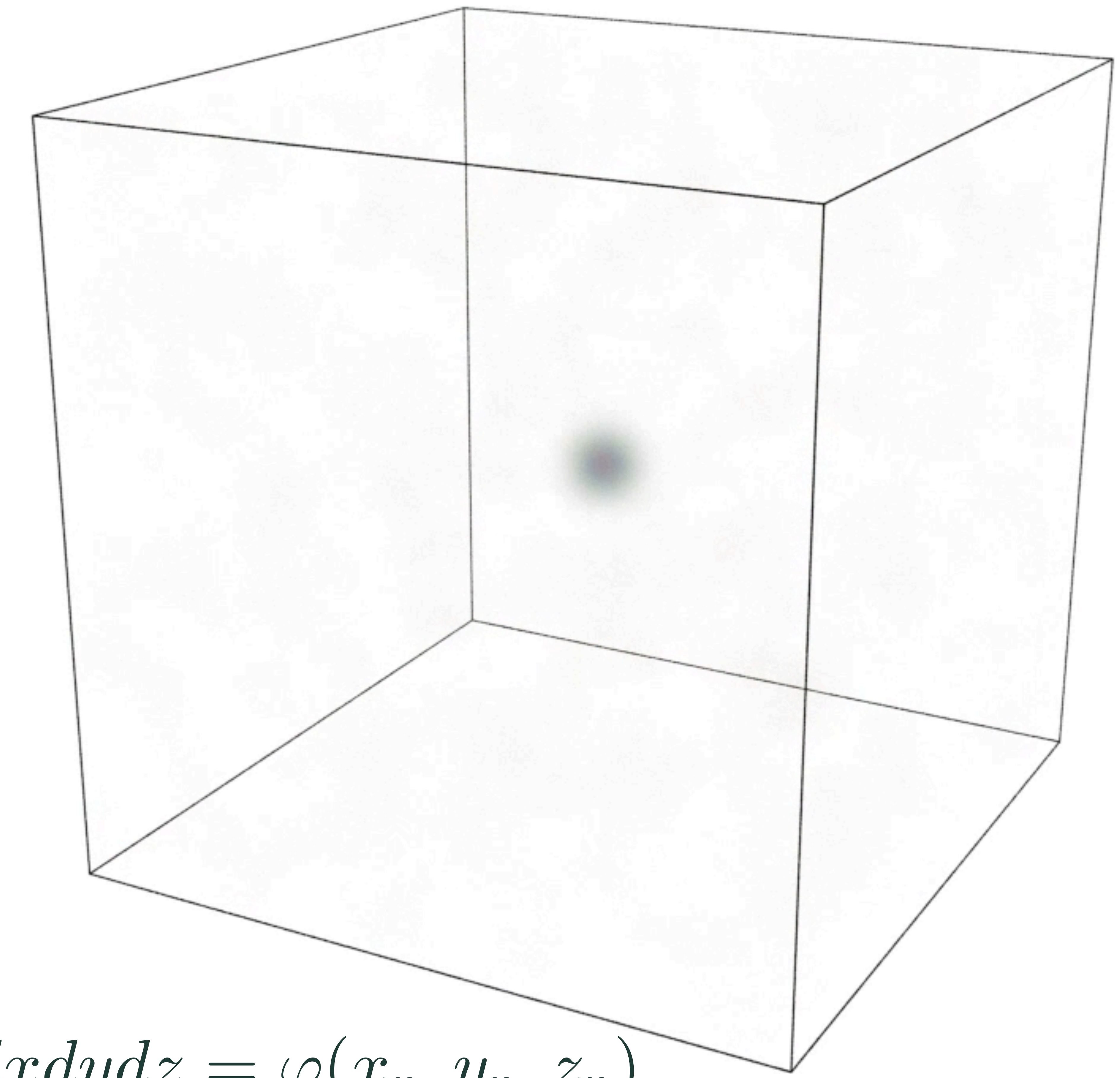
φ: a test function (0-form)

$$\int_M \varphi(x,y,z)\delta_p(x,y,z)\,dxdydz = \varphi(x_p, y_p, z_p)$$

3-forms as measures

$\delta_p(x, y, z)\,dx\,dy\,dz$ Dirac delta 3-form

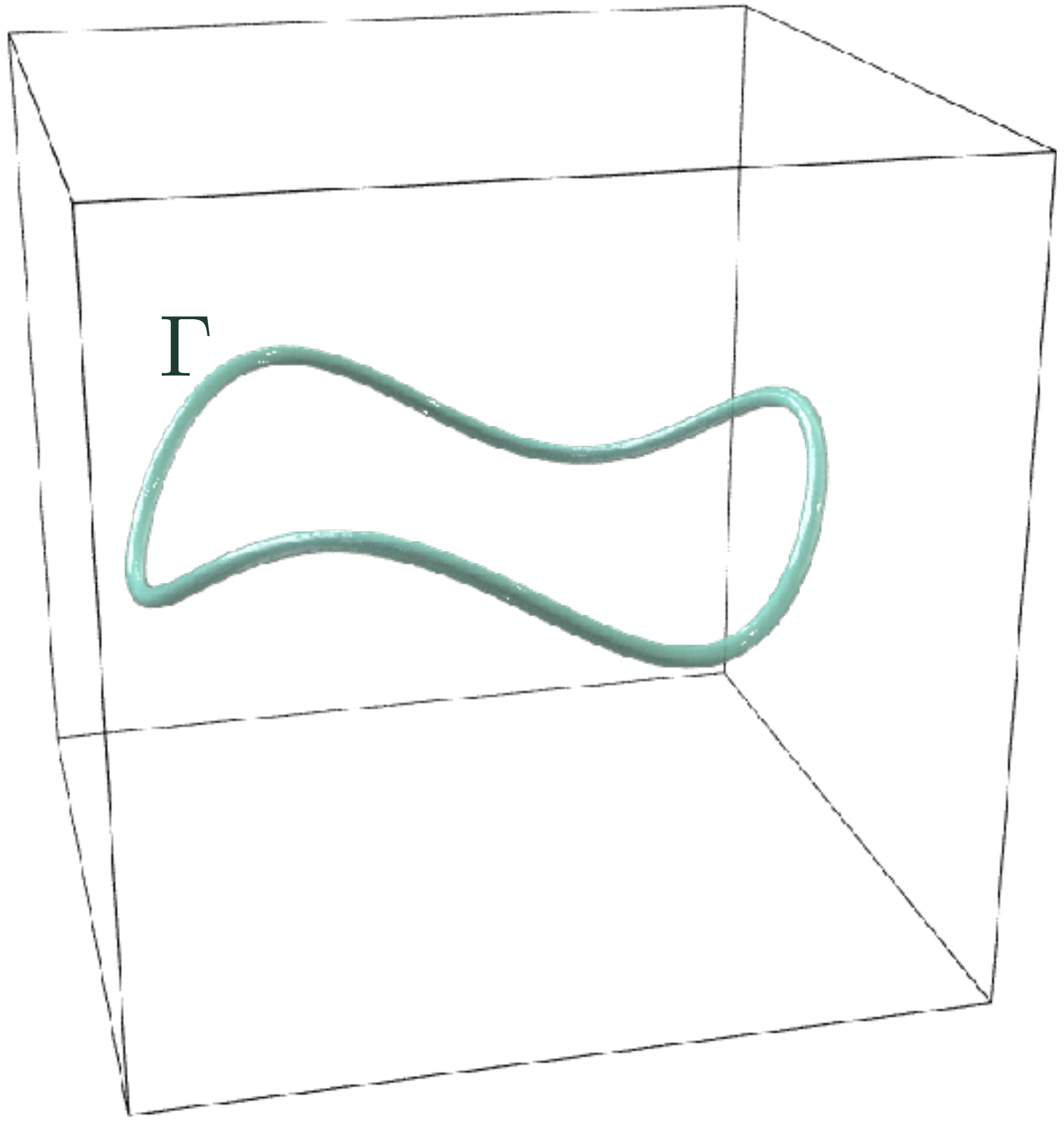
$\delta_p : \varphi \mapsto \varphi(p)$

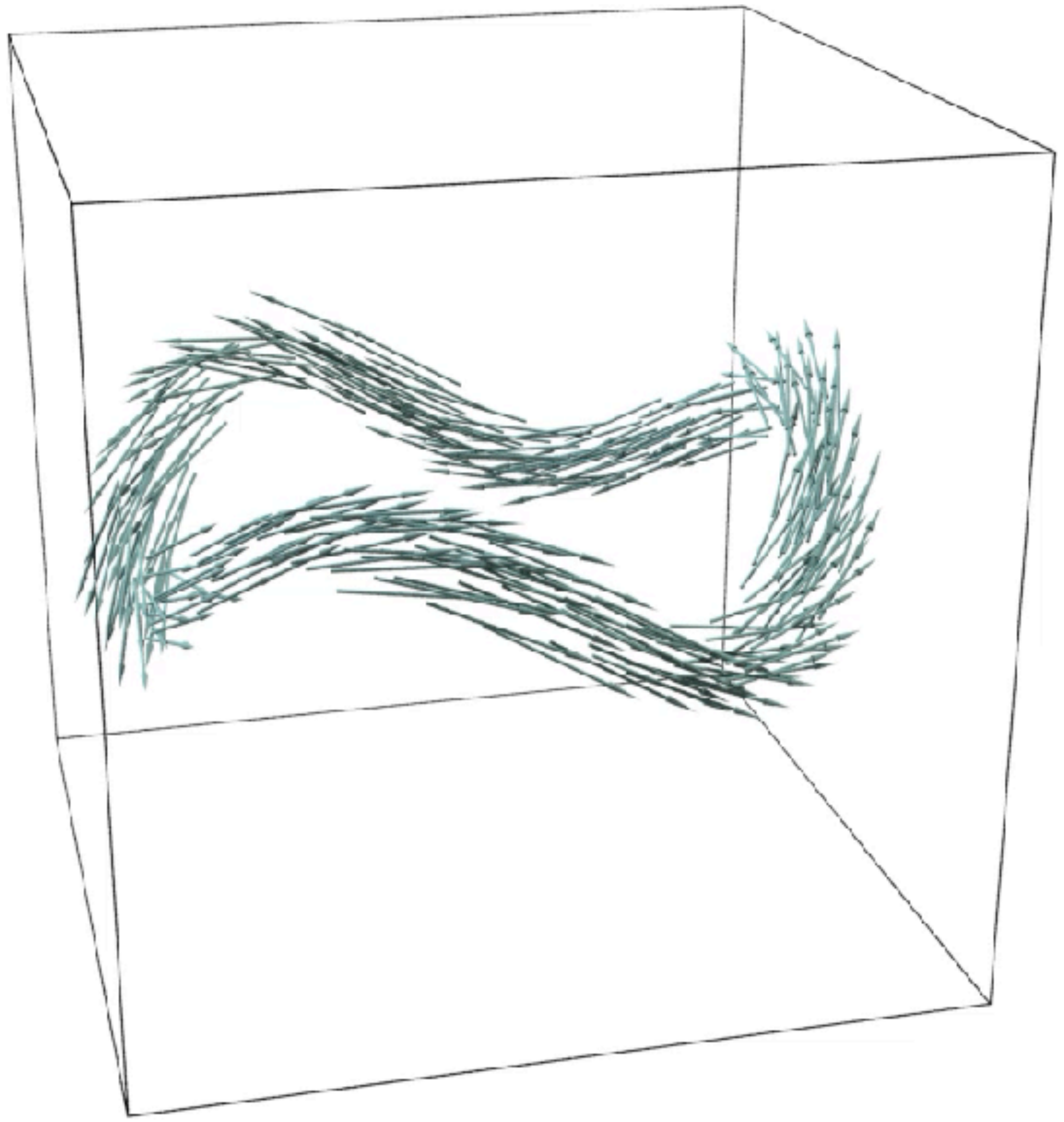


φ: a test function (0-form)

$$\int_M \varphi(x,y,z)\delta_p(x,y,z)dxdydz = \varphi(x_p, y_p, z_p)$$

2-forms as curves





$\delta_p : \varphi \mapsto \varphi(p)$ Dirac-delta 3-form

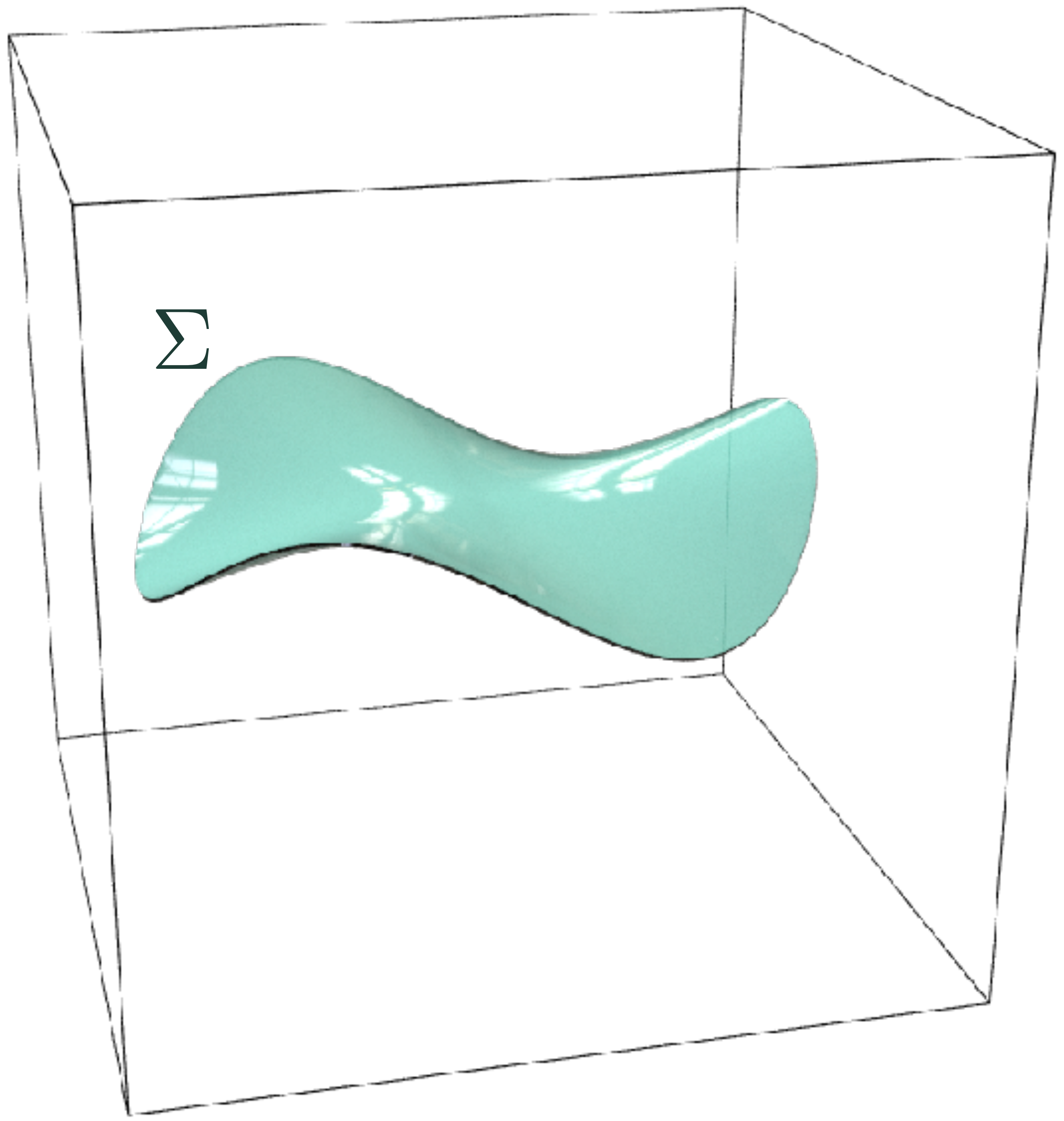
$\eta = f\,dx + g\,dy + h\,dz$: a test 1-form

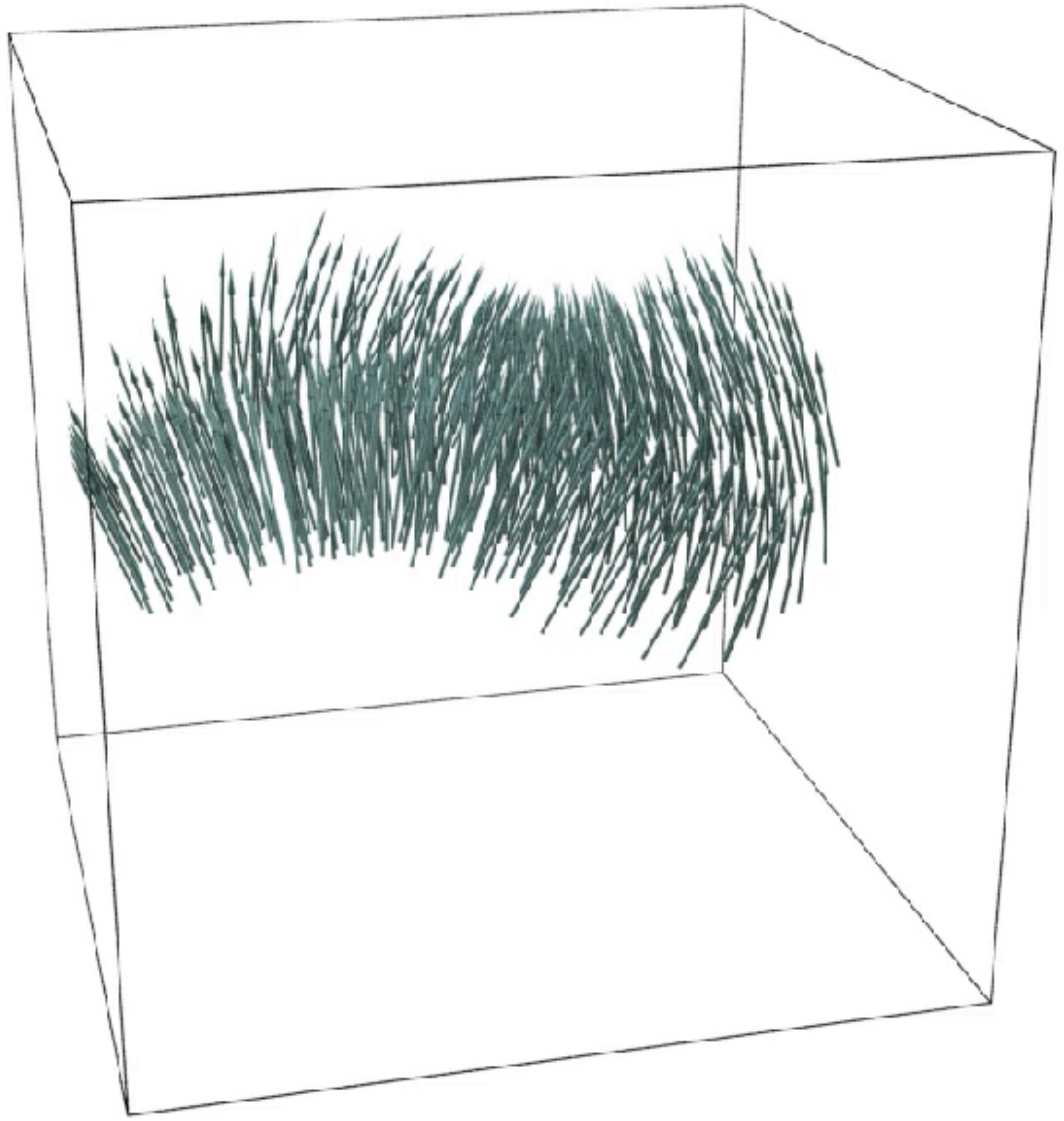
$\delta_\Gamma : \eta \mapsto \int_\Gamma \eta$ Dirac-delta 2-form of a curve Γ

$\eta = f\,dx + g\,dy + h\,dz$: a test 1-form

$$\int_M \eta \wedge \delta_\Gamma = \int_\Gamma \eta$$

1-forms as surfaces





$\varphi \mapsto \varphi(x_p, y_p, z_p)$ Dirac-delta 3-form

$\delta_\Gamma : \eta \mapsto \int_\Gamma \eta$ Dirac-delta 2-form of a curve Γ

$\omega = F dydz + G dzdx + H dxdy$: a test 2-form

$\delta_\Sigma : \omega \mapsto \int_\Sigma \omega$ Dirac-delta form of a surface Σ

$\omega = F dydz + G dzdx + H dxdy$: a test 2-form

$$\int_M \omega \wedge \delta_\Sigma = \int_\Sigma \omega$$

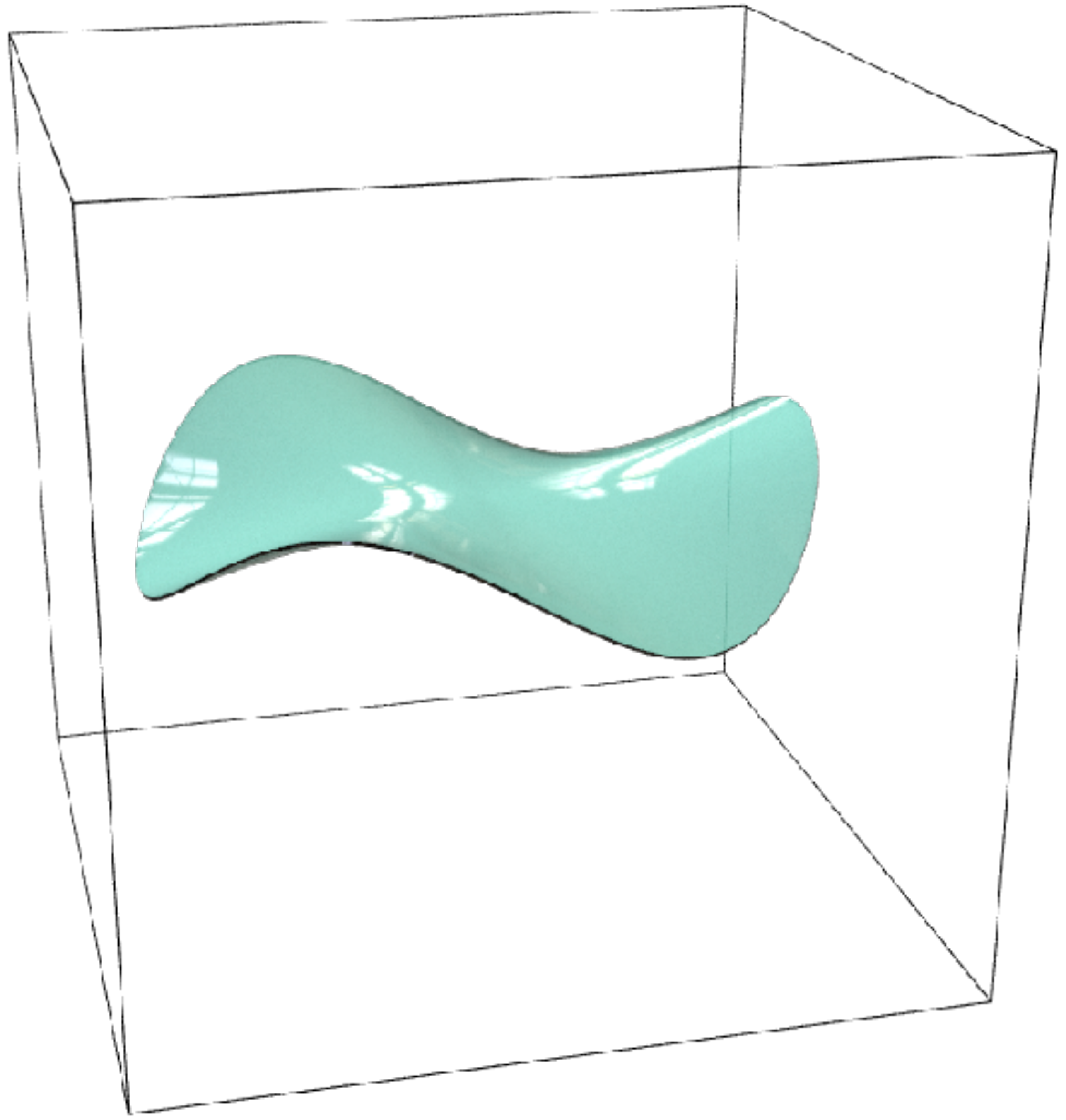
Plateau problem

(the problem of minimal surfaces)

Variable Σ

Constraint $\partial\Sigma = \Gamma$ a given space curve

Objective $\text{Area}(\Sigma)$



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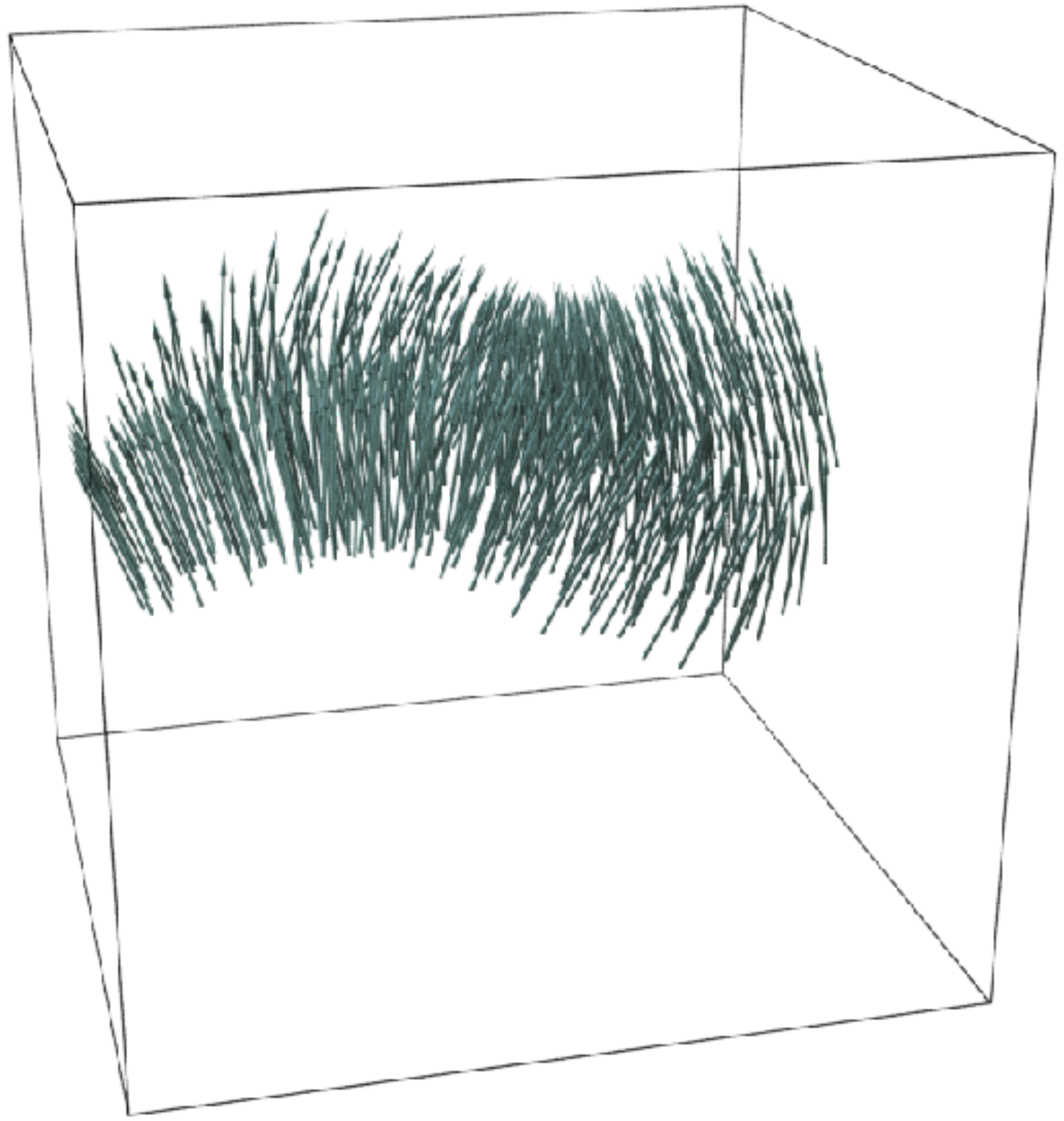
Plateau problem

(the problem of minimal surfaces)

Variable δ_Σ

Constraint $\partial\Sigma = \Gamma$ (a given space curve)

Objective $\mathrm{Area}(\Sigma)$



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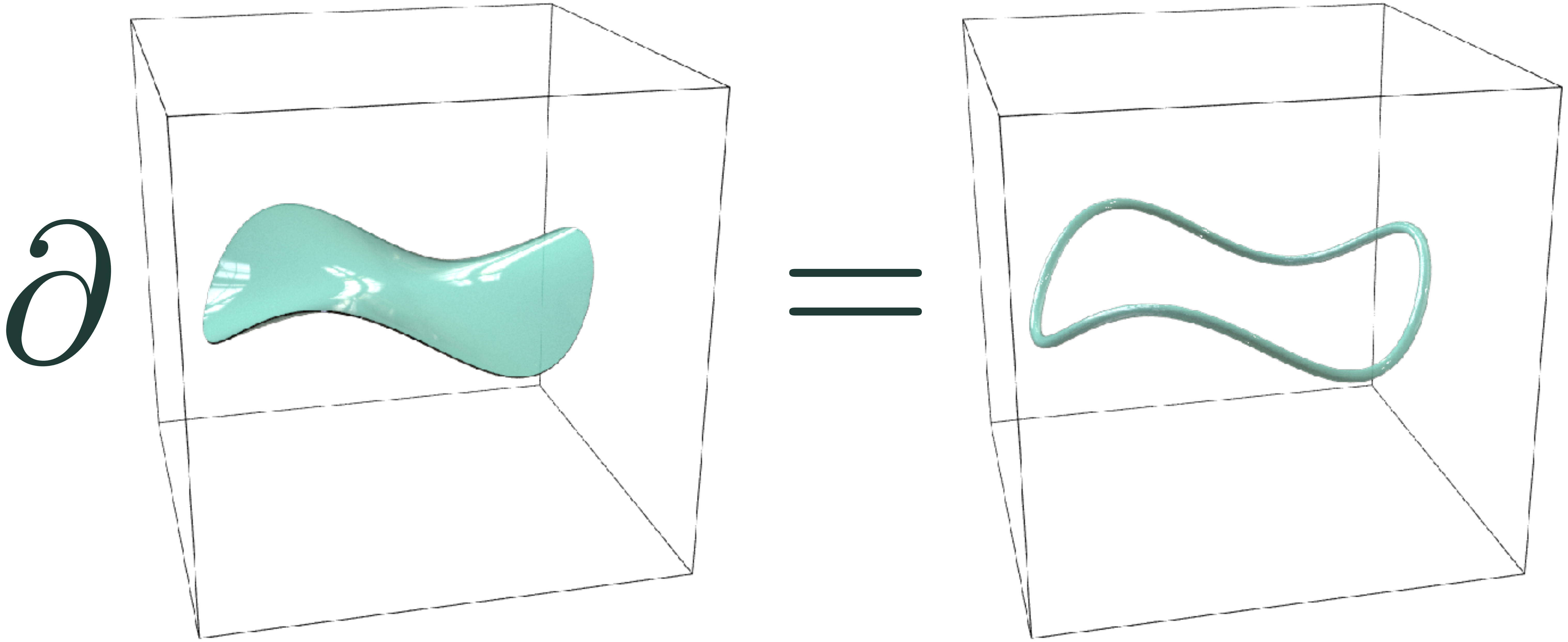
$$\partial\Sigma = \Gamma$$

the boundary constraint



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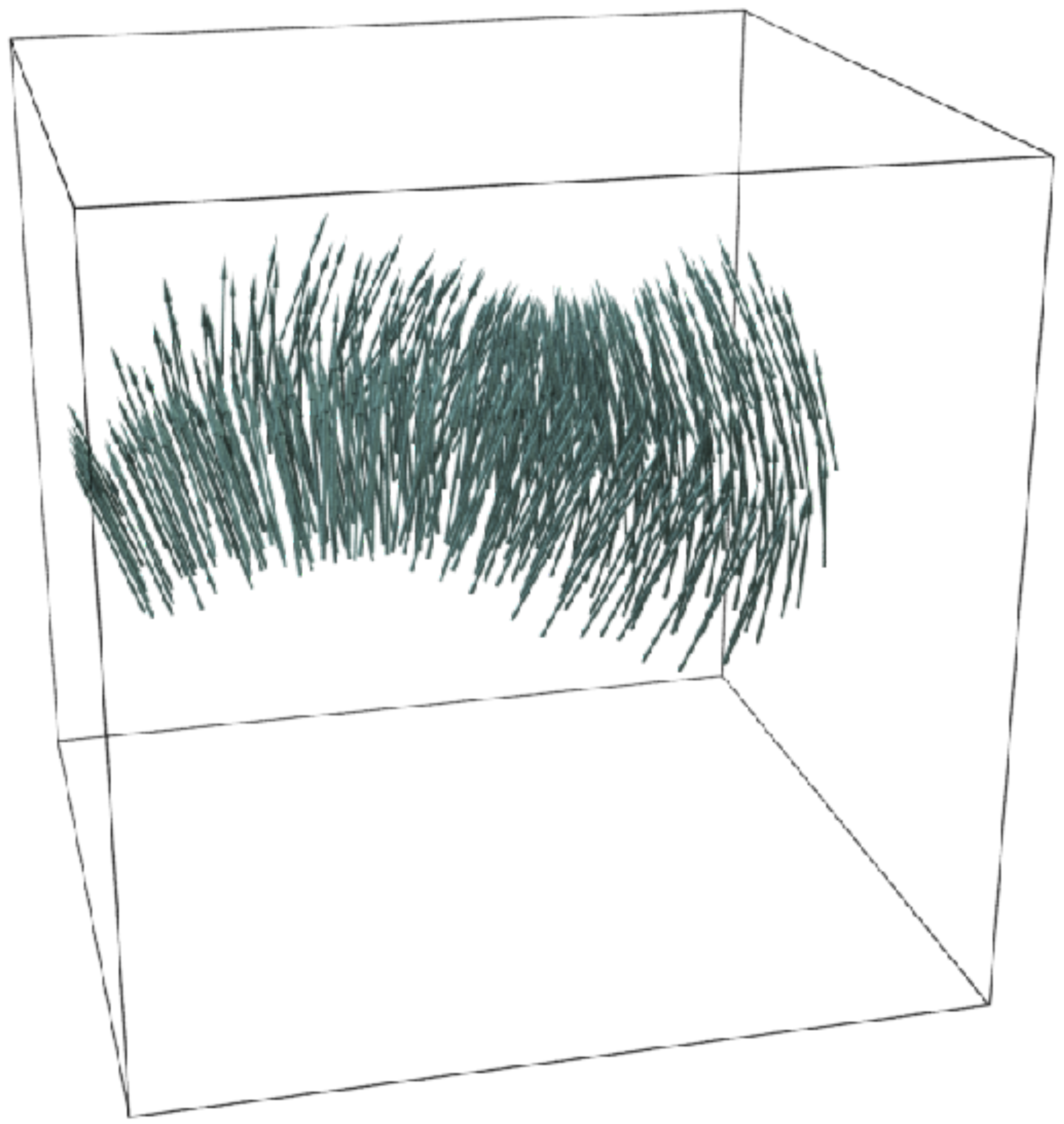
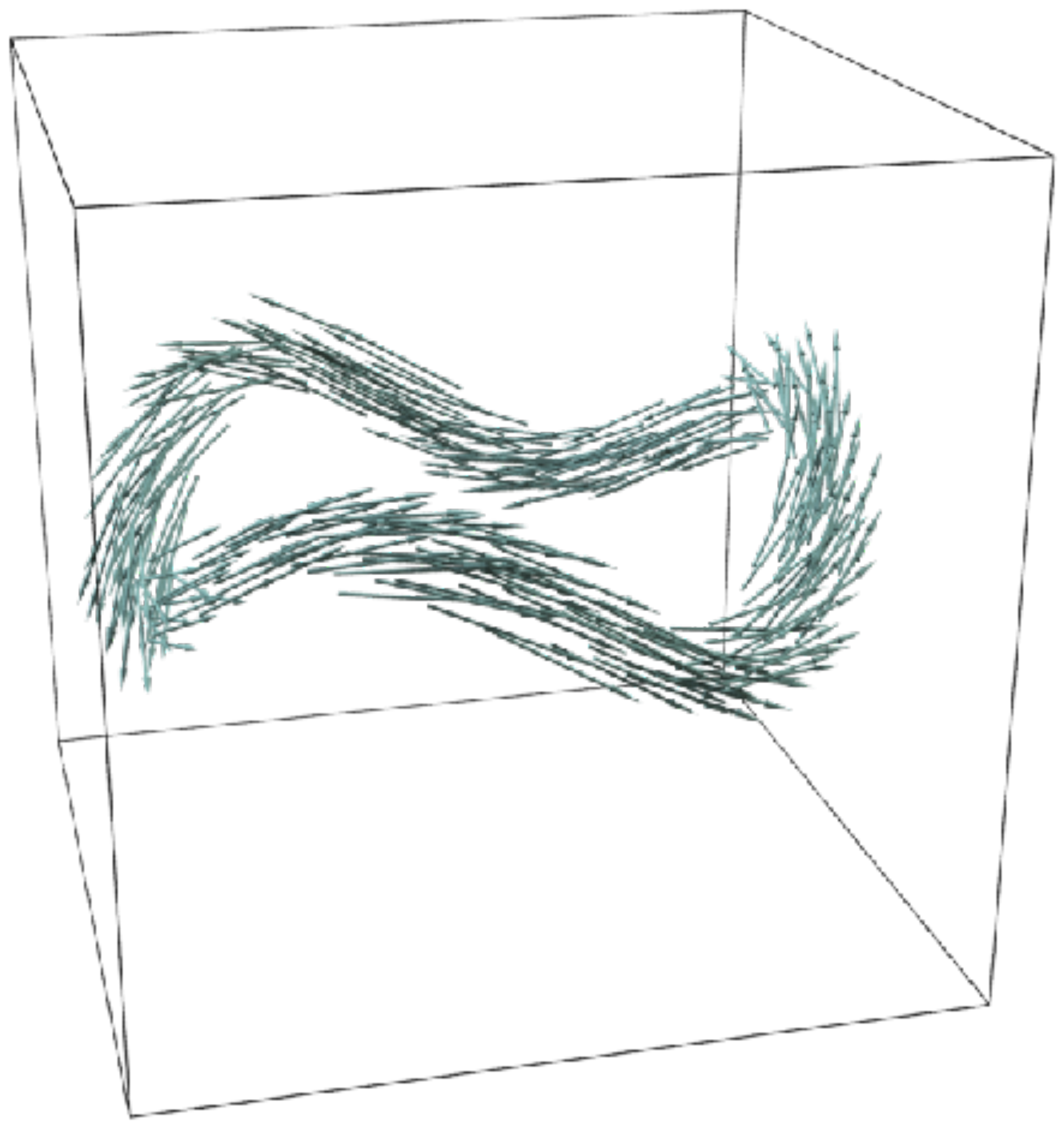
$$\partial\Sigma = \Gamma$$

$$d\delta_\Sigma = \delta_\Gamma$$

the boundary constraint

d  $=$ 

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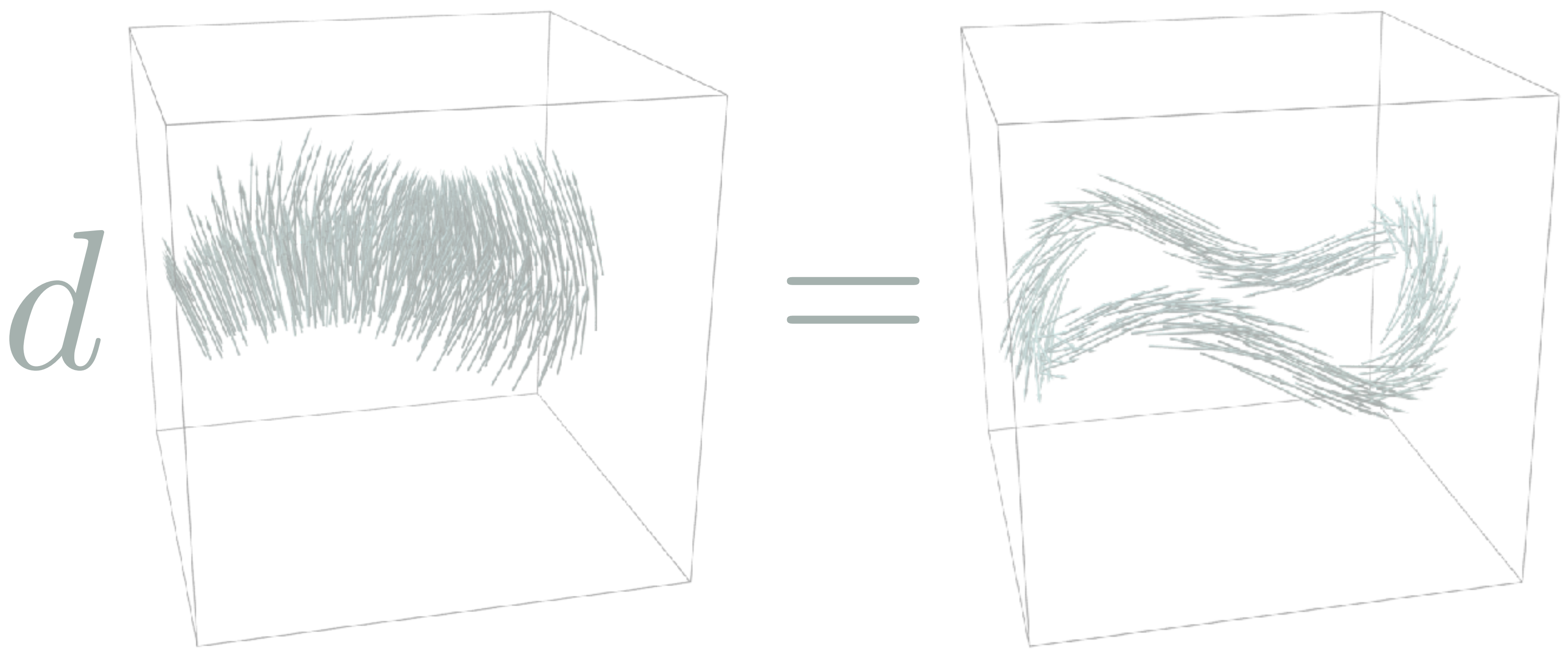
$$\partial\Sigma = \Gamma$$

$$d\delta_\Sigma = \delta_\Gamma$$

the boundary constraint

- Stokes' Theorem $\int_{\partial N} \nu = \int_N d\nu$
- In other words, $\delta_{\partial N}[\nu] = \delta_N \circ d[\nu]$



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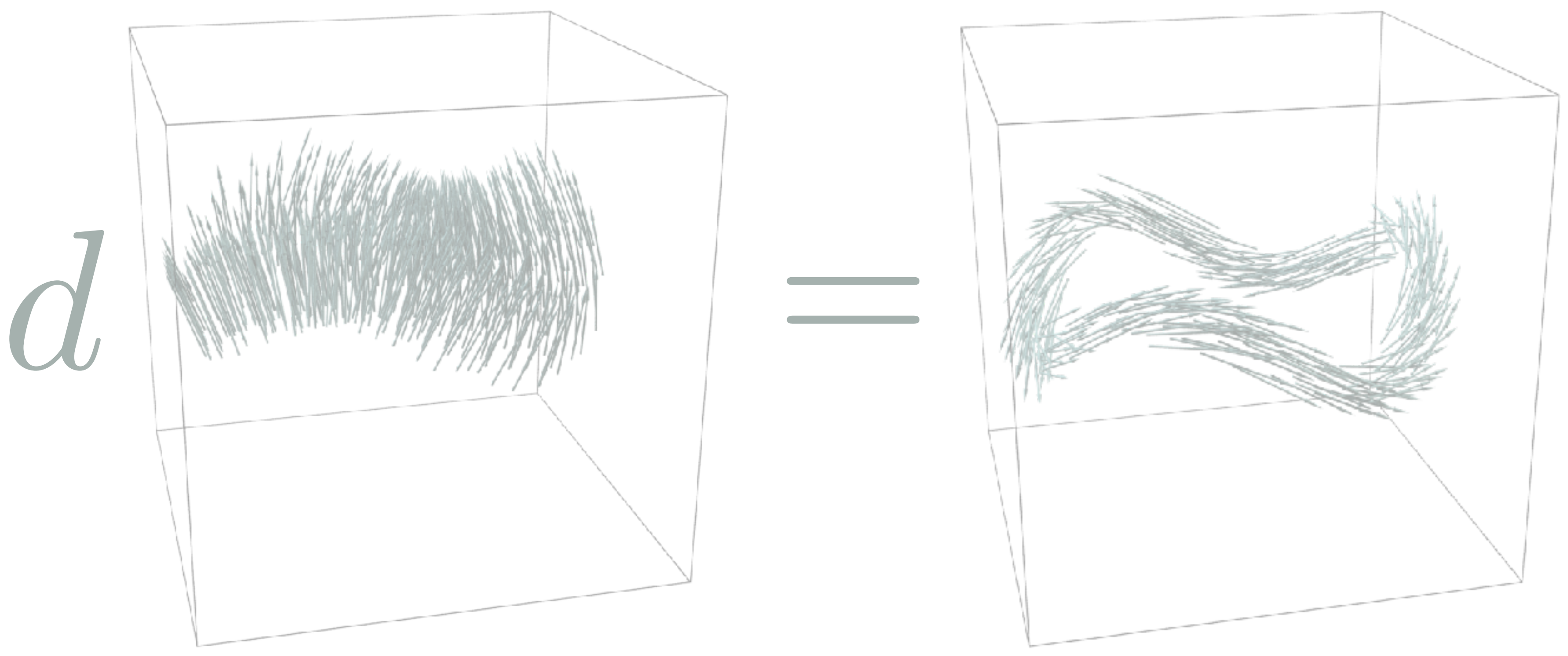
$$\partial\Sigma = \Gamma$$

$$d\delta_{\Sigma} = \delta_{\Gamma}$$

the boundary constraint

- f' is the weak derivative of f if $\forall \varphi \in C^{\infty}, \int \varphi f' = \oint \varphi f - \int \varphi' f$



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$$\partial\Sigma = \Gamma$$

$$d\delta_\Sigma = \delta_\Gamma$$

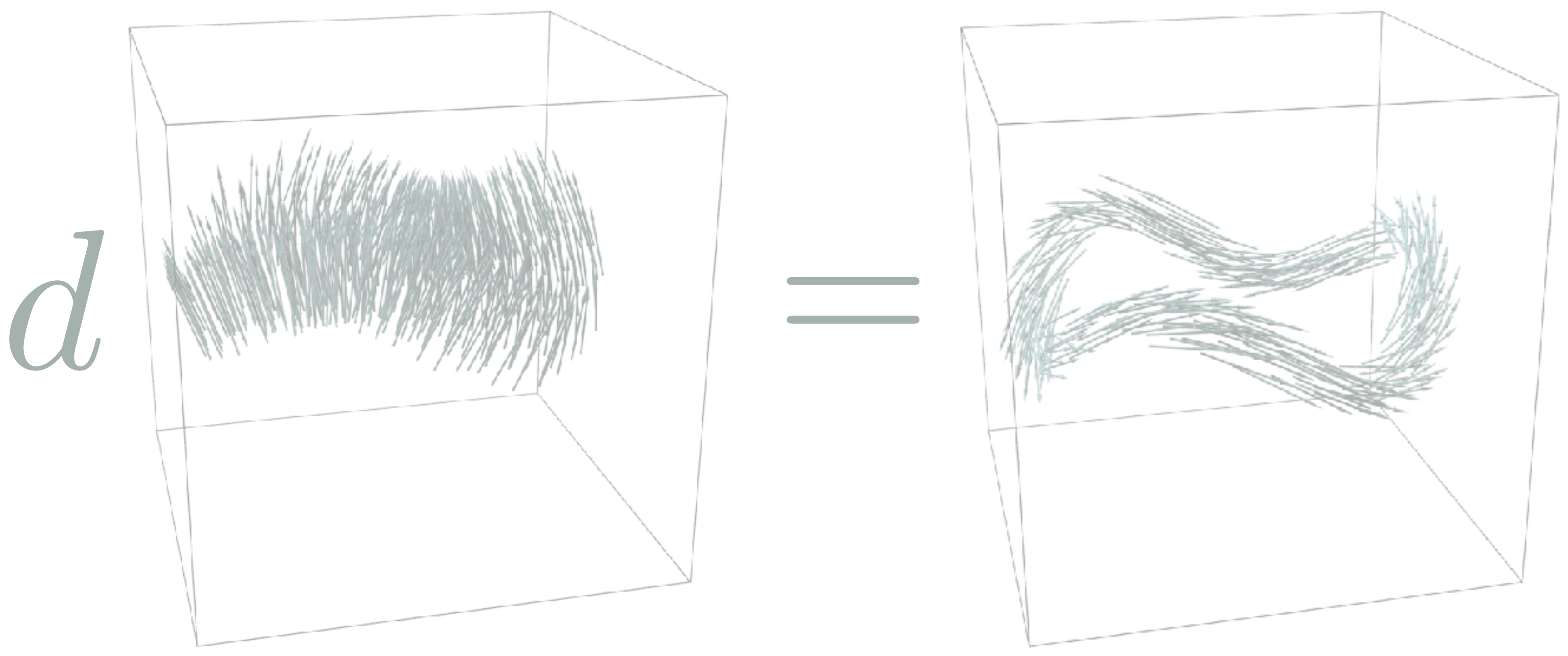
the boundary constraint

- f' is the weak derivative of f if $\forall \varphi \in C^\infty, \int \varphi f' = \oint \varphi f - \int \varphi' f$

- $d\eta$ is the weak derivative of $\eta \in \Omega^{n-k}$ if

$$\forall \omega \in C^\infty \Omega^{k-1}(M), (-1)^{k-1} \int_M \omega \wedge d\eta = \oint_{\partial M} \omega \wedge \eta - \int_M d\omega \wedge \eta$$

an arbitrary smooth (k-1) form



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$$\partial\Sigma = \Gamma$$

$$d\delta_\Sigma = \delta_\Gamma$$

the boundary constraint

- $d\eta$ is the weak derivative of $\eta \in \Omega^{n-k}$ if

$$\forall \omega \in C^\infty\Omega^{k-1}(M), (-1)^{k-1}\int_M \omega \wedge d\eta = \oint_{\partial M} \omega \wedge \eta - \int_M d\omega \wedge \eta$$

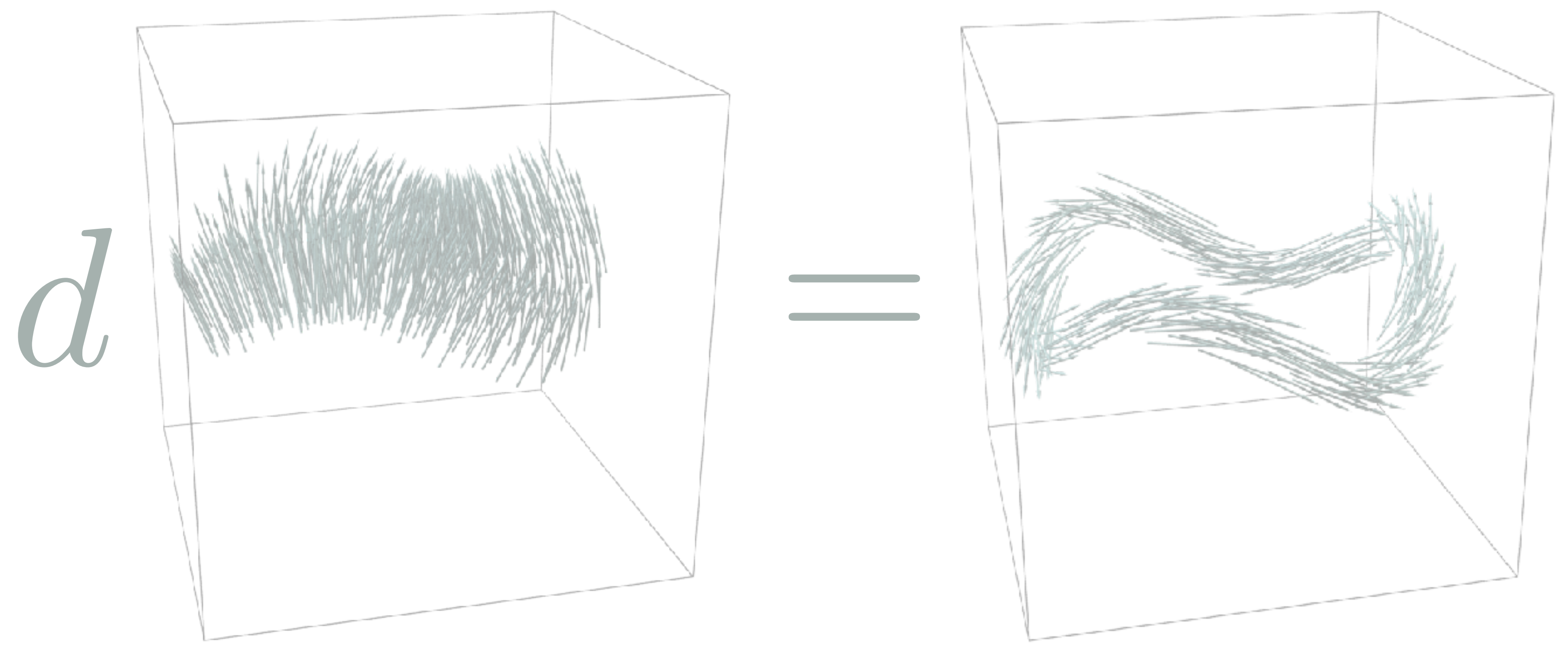
an arbitrary smooth (k-1) form

- For Dirac-delta 1-form $\eta = \delta_\Sigma$,

$$(-1)\int_M \omega \wedge d\delta_\Sigma = \oint_{\partial M} \omega \wedge \delta_\Sigma - \int_M d\omega \wedge \delta_\Sigma$$

$$= -\int_M d\omega \wedge \delta_\Sigma$$

$$= -\oint_{\partial\Sigma} \omega = -\int_M \omega \wedge \delta_{\partial\Sigma}$$



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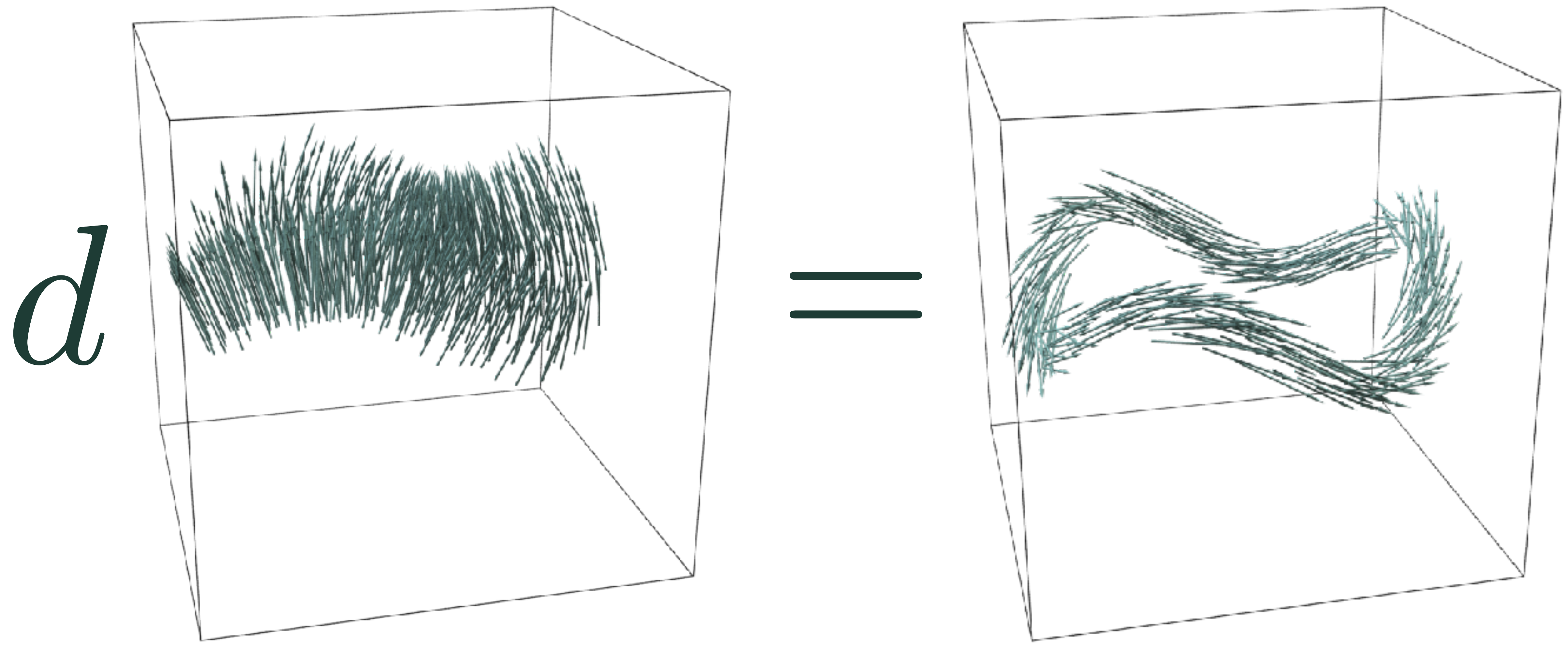
Plateau problem

(the problem of minimal surfaces)

Variable	δ_Σ	
Constraint	$\partial\Sigma = \Gamma$	a given space curve
Objective	$\mathrm{Area}(\Sigma)$	



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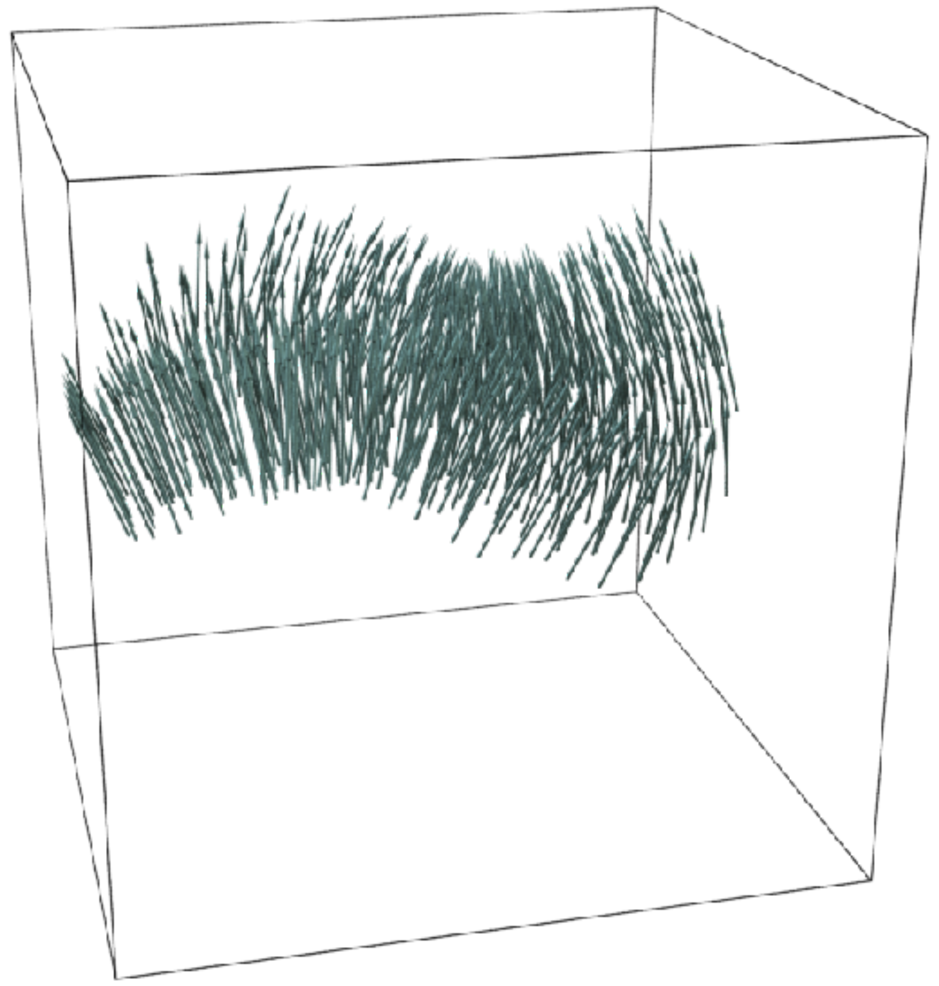
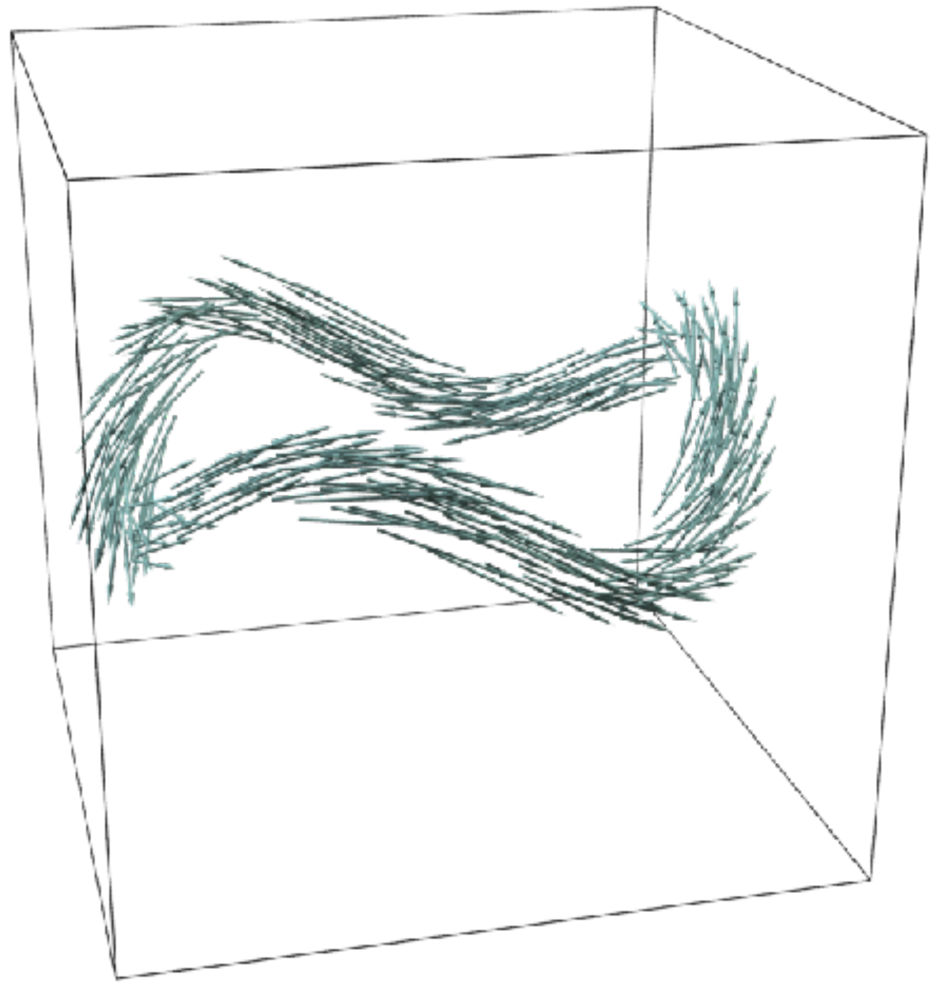
Plateau problem

(the problem of minimal surfaces)

Variable	δ_Σ
Constraint	$d\delta_\Sigma = \delta_\Gamma$ (a given space curve)
Objective	$\text{Area}(\Sigma)$

d  $=$ 

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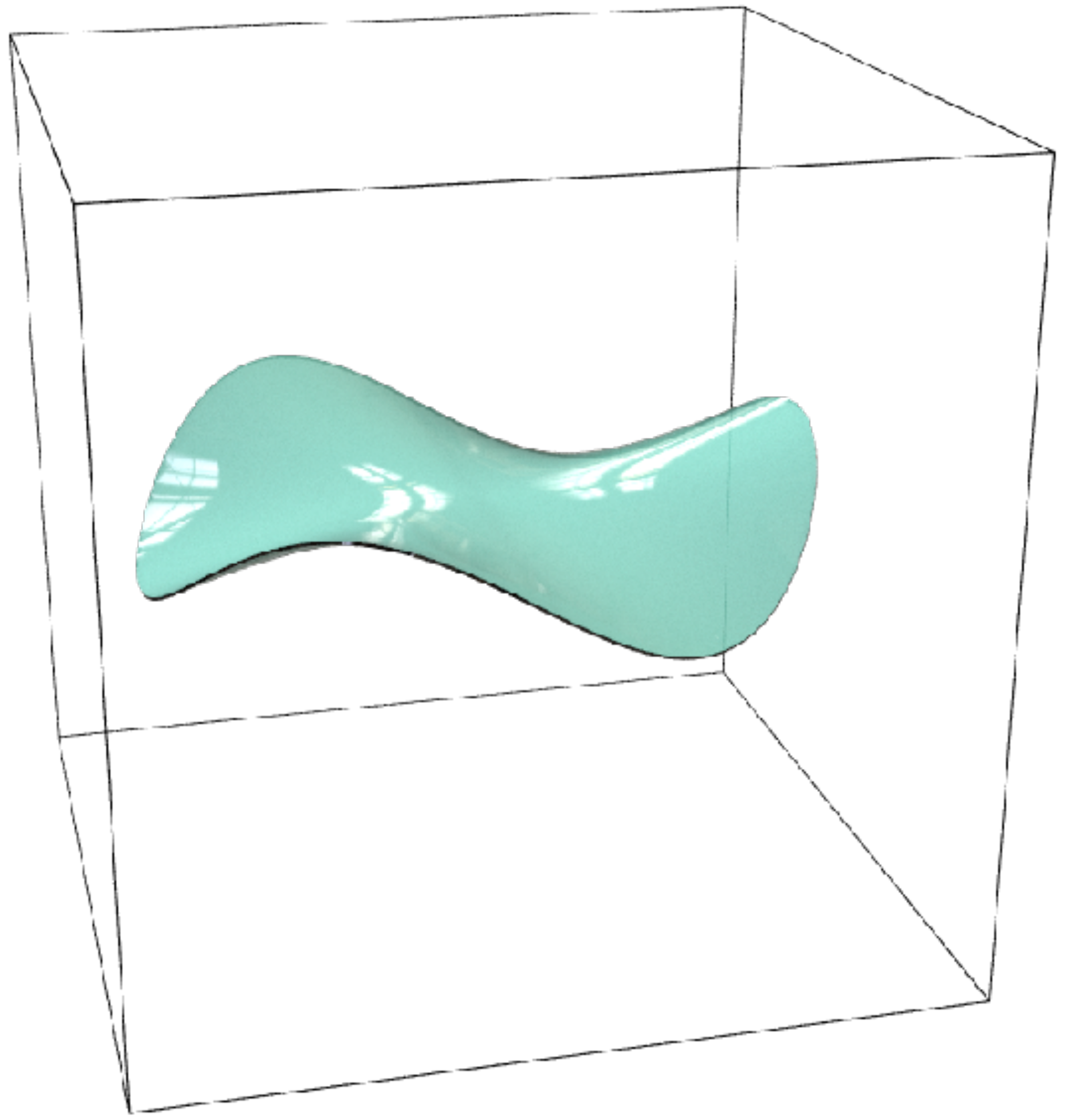
$$\mathrm{Area}(\Sigma)$$

the area functional

Area()

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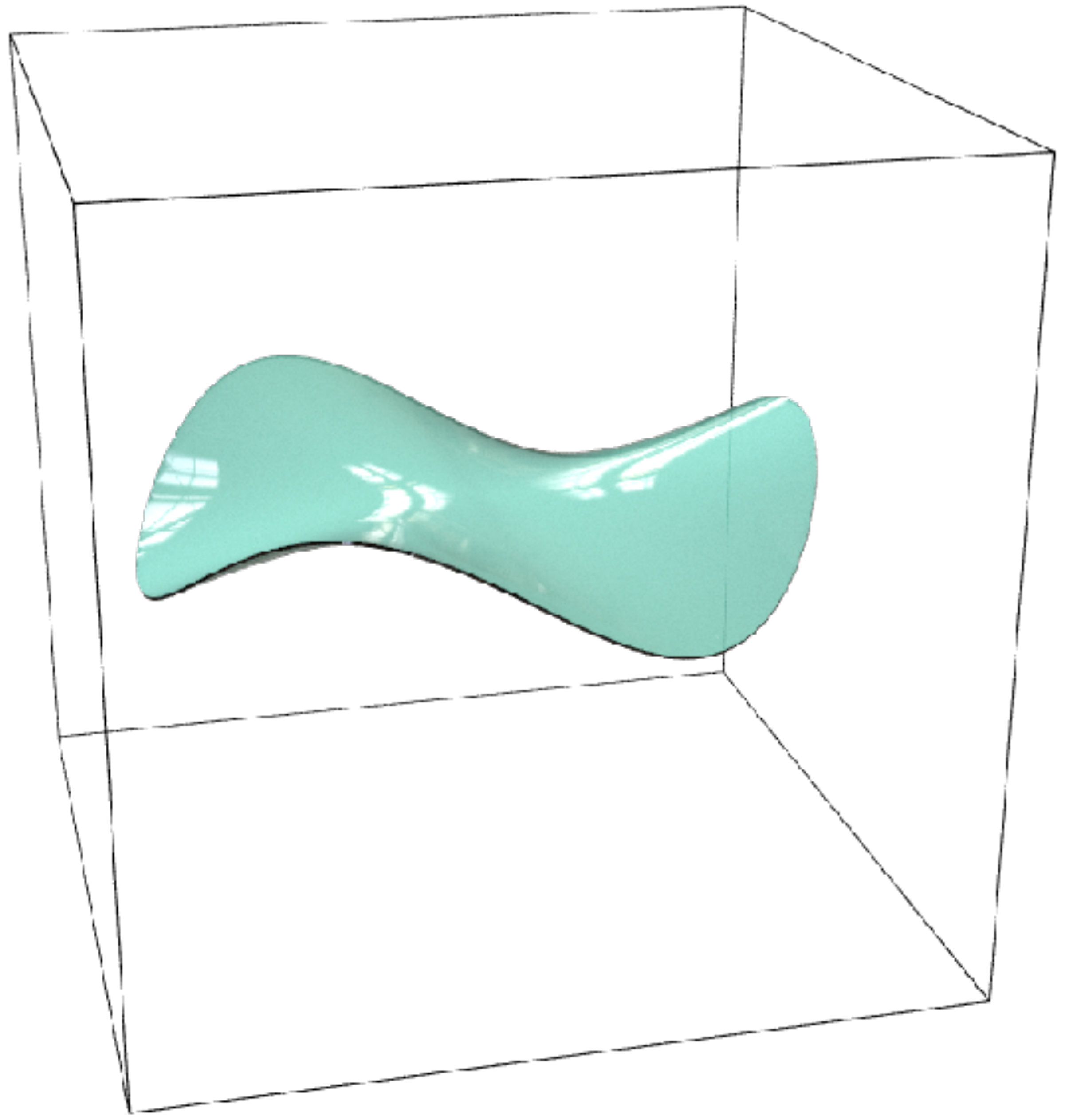
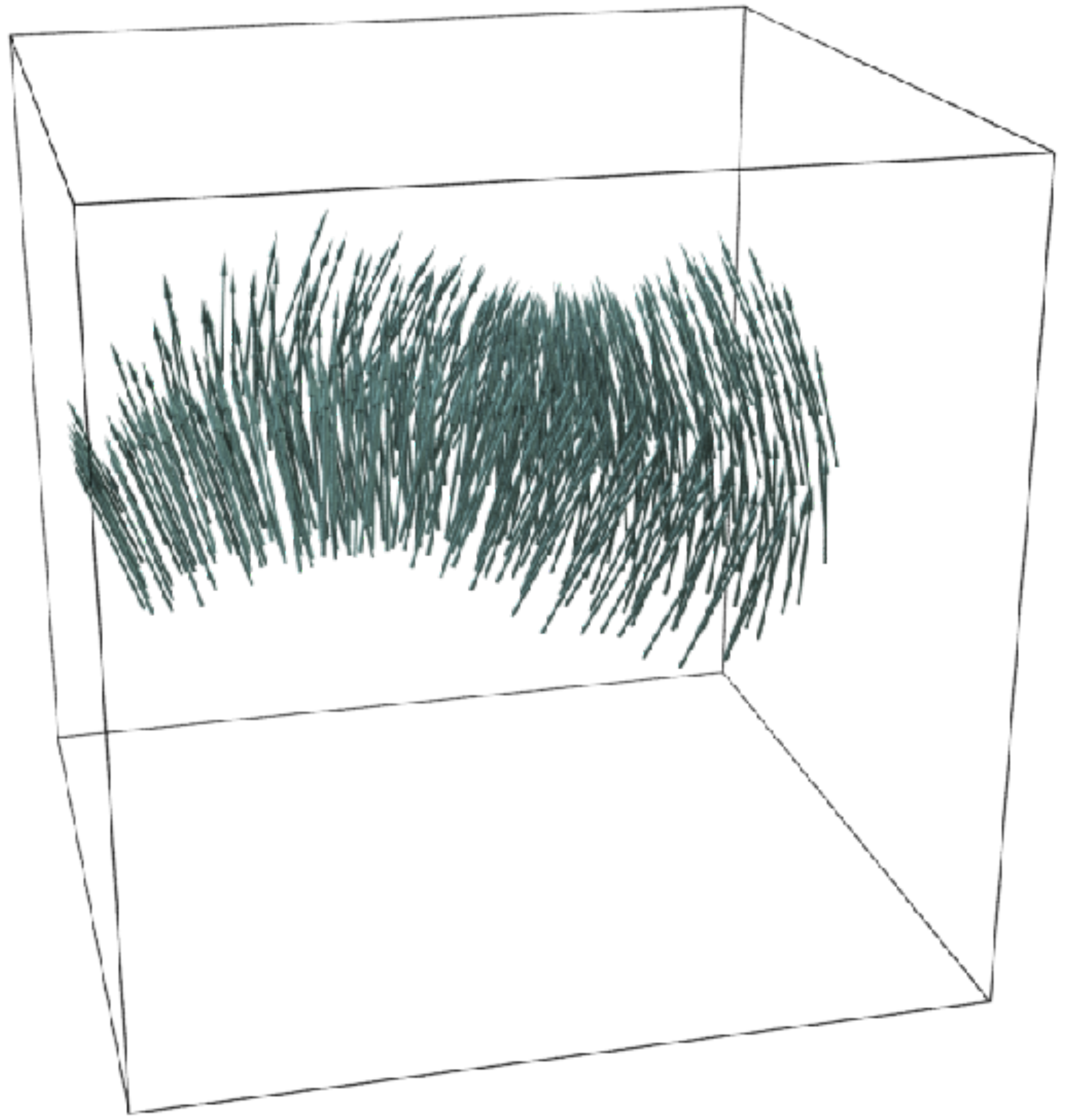
$$\mathrm{Area}(\Sigma) = \|\delta_\Sigma\|_{\mathrm{mass}}$$

the area functional

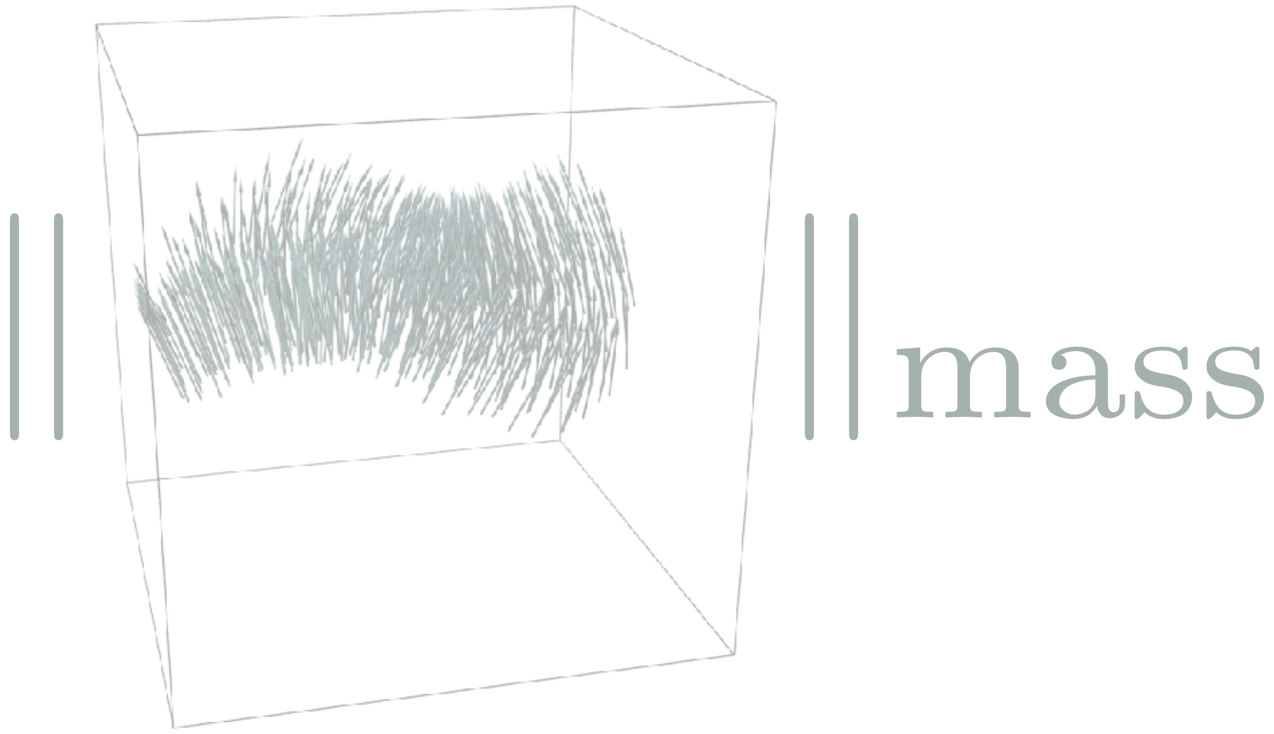
Area() = ||  $\|_{\mathrm{mass}}$

$$\text{Area}(\Sigma) = \|\delta_\Sigma\|_{\text{mass}}$$

the area functional = operator norm w.r.t. sup-norm

- Operator norm for (n-k)-form $\eta : C^\infty\Omega^k(M) \to \mathbb{R}$

$$\|\eta\|_{\text{mass}} = \sup_{\omega \in C^\infty\Omega^k(M),\, \|\omega\|_{L^\infty} \leq 1} |\eta[\omega]|$$



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$$\mathrm{Area}(\Sigma) = \|\delta_\Sigma\|_{\mathrm{mass}}$$

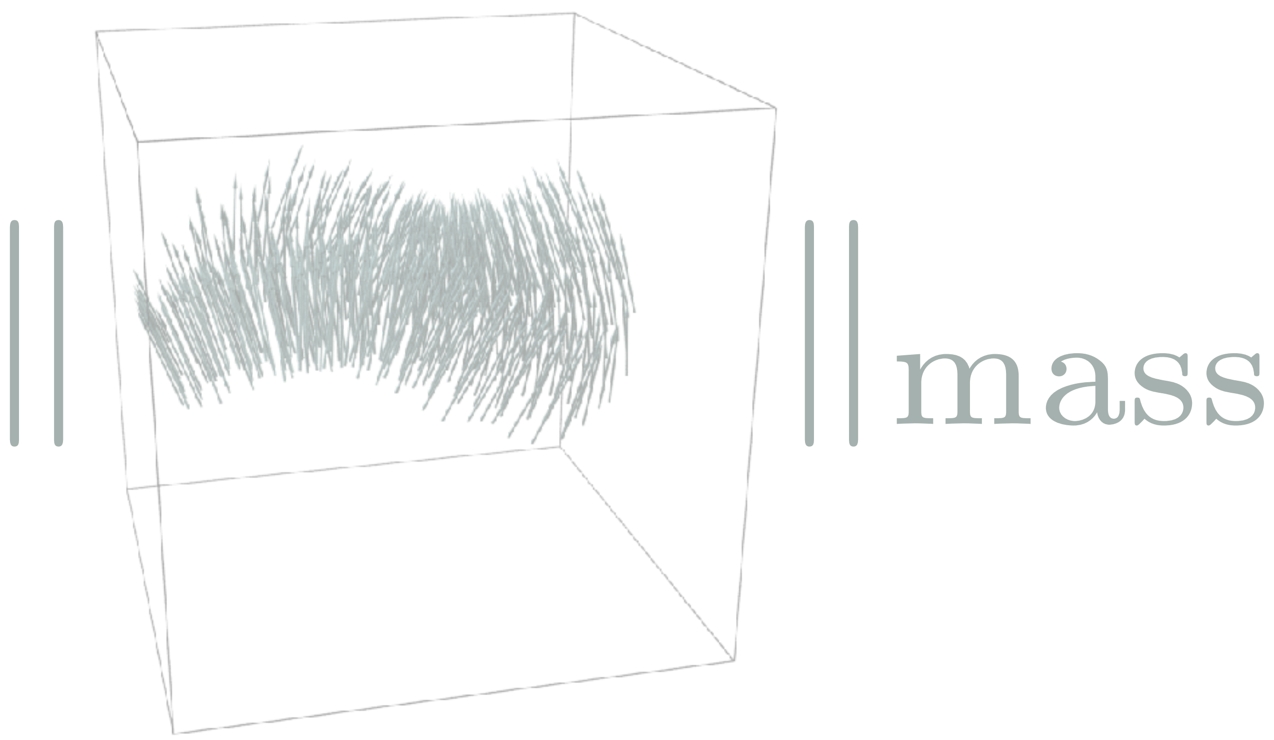
the area functional = operator norm w.r.t. sup-norm

- Operator norm for (n-k)-form $\eta : C^\infty\Omega^k(M) \to \mathbb{R}$

$$\|\eta\|_{\mathrm{mass}} = \sup_{\omega \in C^\infty\Omega^k(M), \|\omega\|_{L^\infty} \leq 1} |\eta[\omega]|$$

- Sup-norm for k-smooth form $\omega \in C^\infty\Omega^k(M)$

$$\|\omega\|_{L^\infty} = \sup_{p \in M} |\omega|_p$$



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$$\mathrm{Area}(\Sigma) = \|\delta_\Sigma\|_{\mathrm{mass}}$$

the area functional = operator norm w.r.t. sup-norm

- Operator norm for (n-k)-form $\eta : C^\infty\Omega^k(M) \to \mathbb{R}$

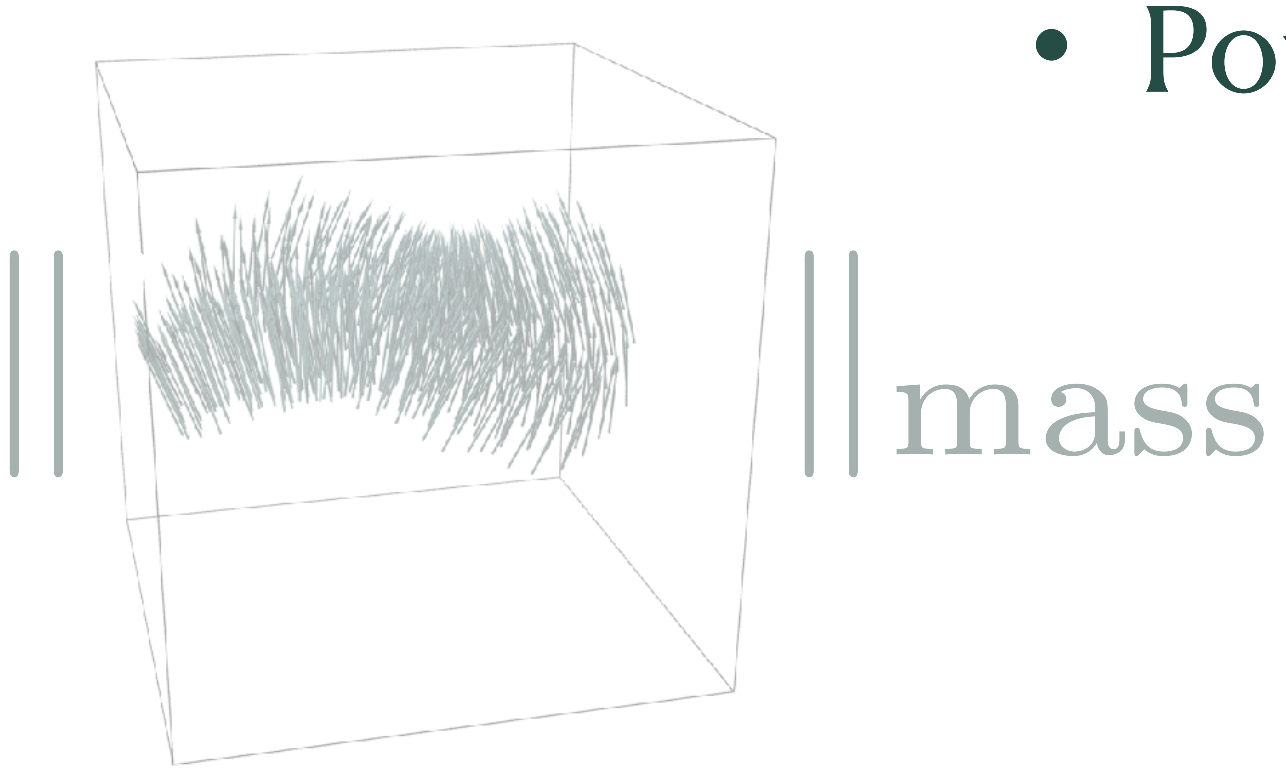
$$\|\eta\|_{\mathrm{mass}} = \sup_{\omega \in C^\infty\Omega^k(M), \|\omega\|_{L^\infty} \leq 1} |\eta[\omega]|$$

- Sup-norm for k-smooth form $\omega \in C^\infty\Omega^k(M)$

$$\|\omega\|_{L^\infty} = \sup_{p\in M} |\omega|_p$$

- Pointwise Euclidean ℓ^2 norm

$$|\omega|_p = \sqrt{\star(\omega \wedge \star\omega)_p}$$



$$\omega = f_1 dx_2 dx_3 + f_2 dx_3 dx_1 + f_3 dx_1 dx_2, |\omega| = \sqrt{f_1^2 + f_2^2 + f_3^2}$$

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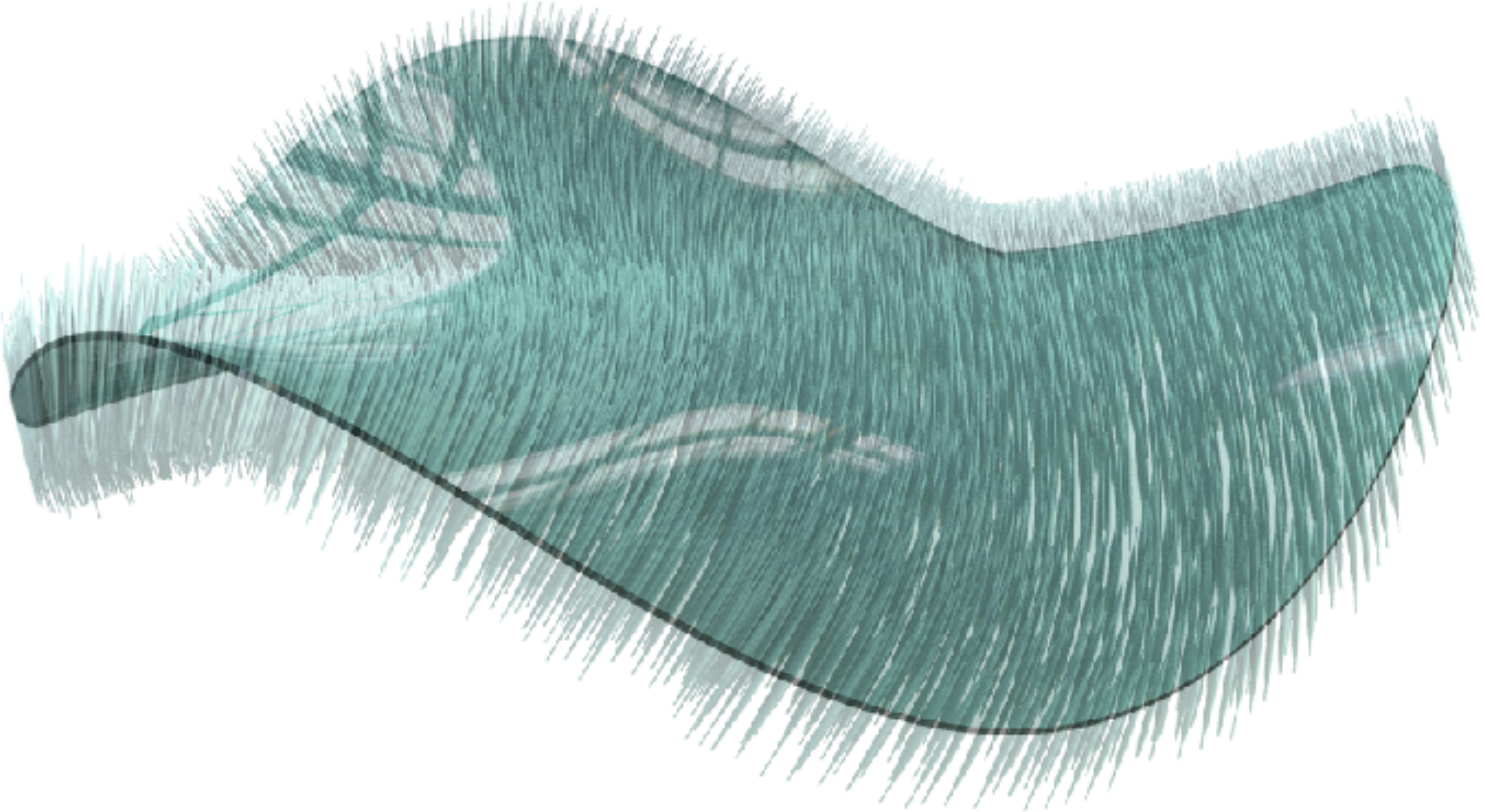
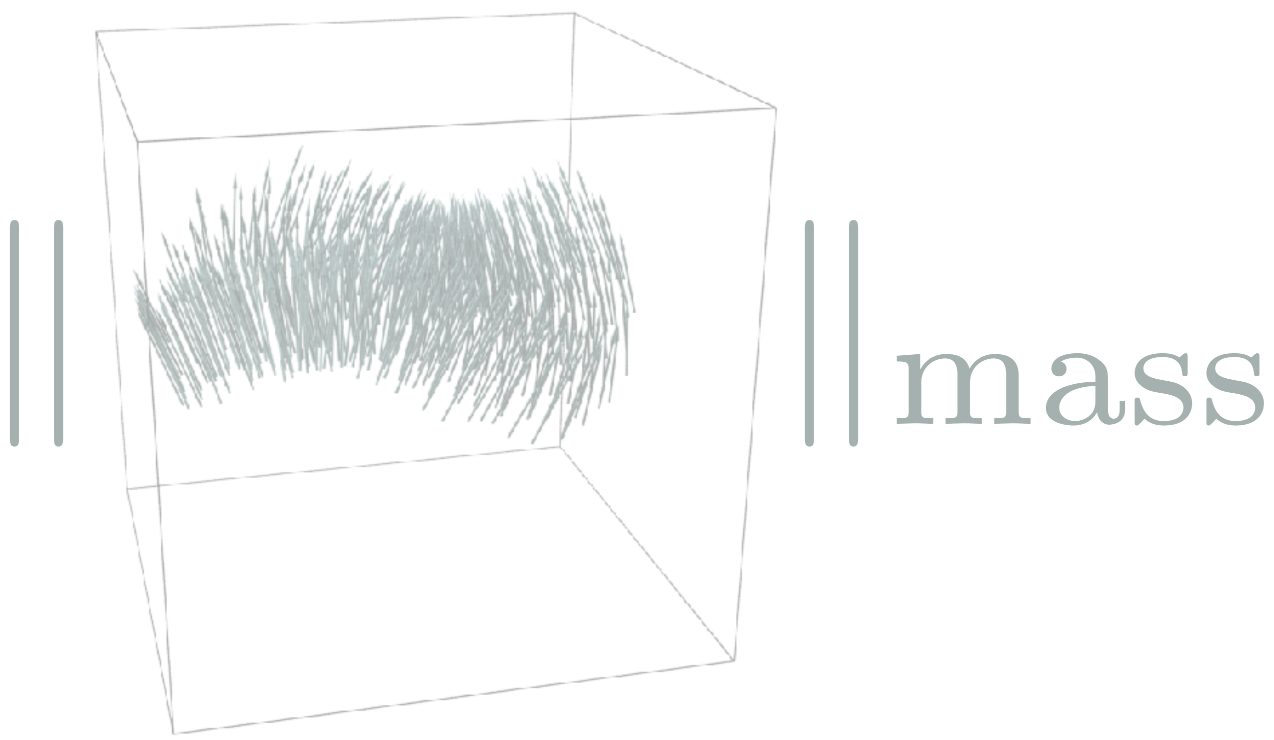
$$\mathrm{Area}(\Sigma) = \|\delta_\Sigma\|_{\mathrm{mass}}$$

the area functional = operator norm w.r.t. sup-norm

- Dirac-delta form for surfaces $\delta_\Sigma^\sharp \approx \frac{1}{\epsilon}\mathbf{n}_\Sigma$



$$\begin{aligned}
\|\delta_\Sigma\|_{\mathrm{mass}} &= \sup_{\substack{\omega \in C^\infty\Omega^k(M) \\ \|\omega\|_{L^\infty} \le 1}} \left| \int_M \omega \wedge \delta_\Sigma \right| \\
&= \sup_{\substack{\mathbf{v} \in C^\infty(M \to \mathbb{R}^3) \\ |\mathbf{v}| \le 1}} \int_\Sigma \mathbf{v} \cdot \mathbf{n}_\Sigma dS \\
&= \int_\Sigma \mathbf{n}_\Sigma \cdot \mathbf{n}_\Sigma dS = \int_\Sigma 1 dS = \mathrm{Area}(\Sigma)
\end{aligned}$$



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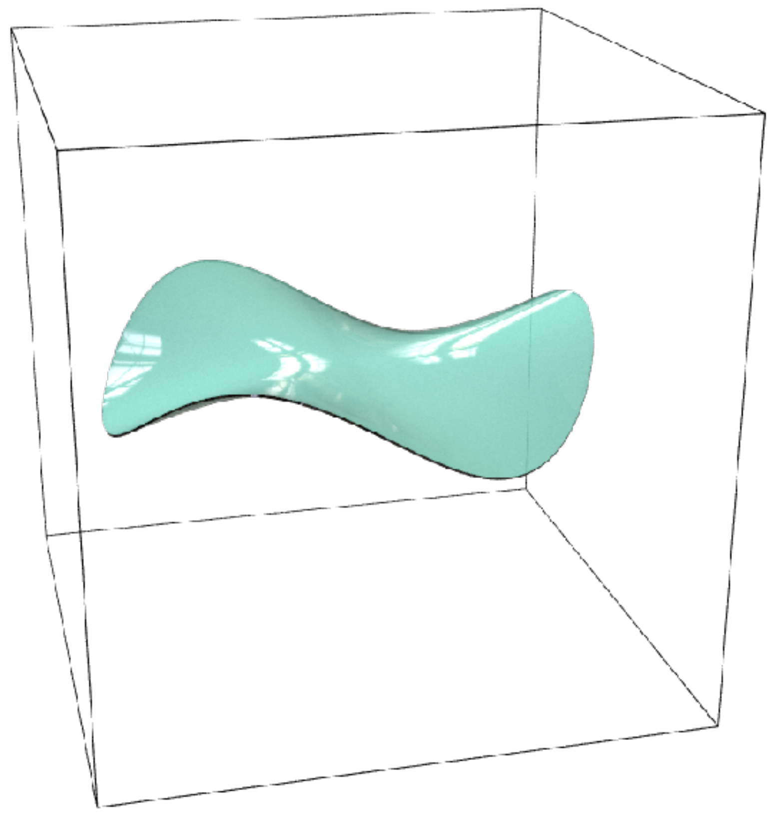
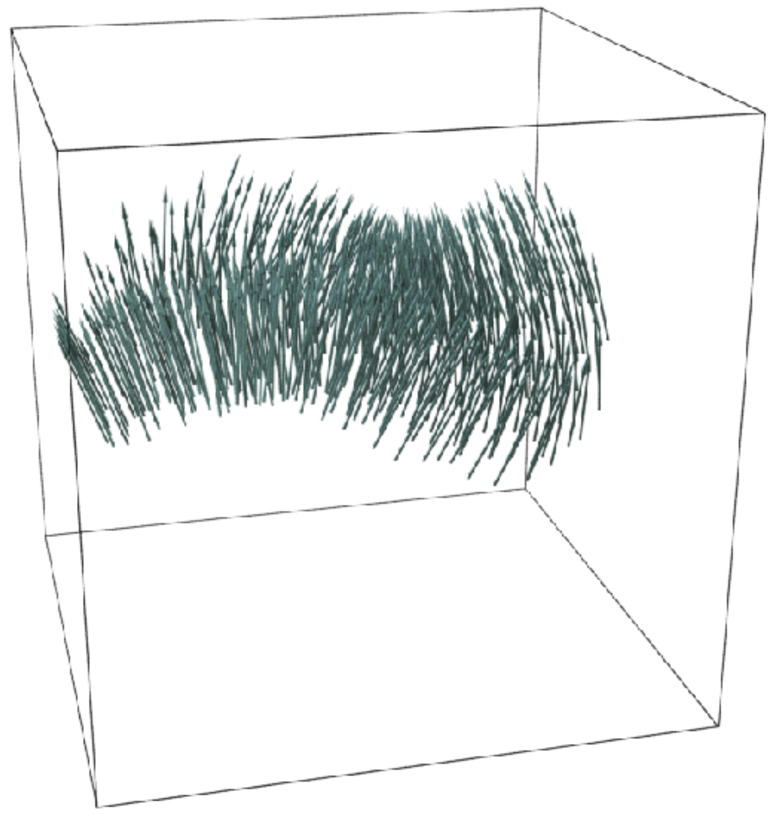
Plateau problem

(the problem of minimal surfaces)

Variable δ_Σ

Constraint $d\delta_\Sigma = \delta_\Gamma$ (a given space curve)

Objective $\boxed{\mathrm{Area}(\Sigma)}$

Area() = ||  ||$_{\mathrm{mass}}$

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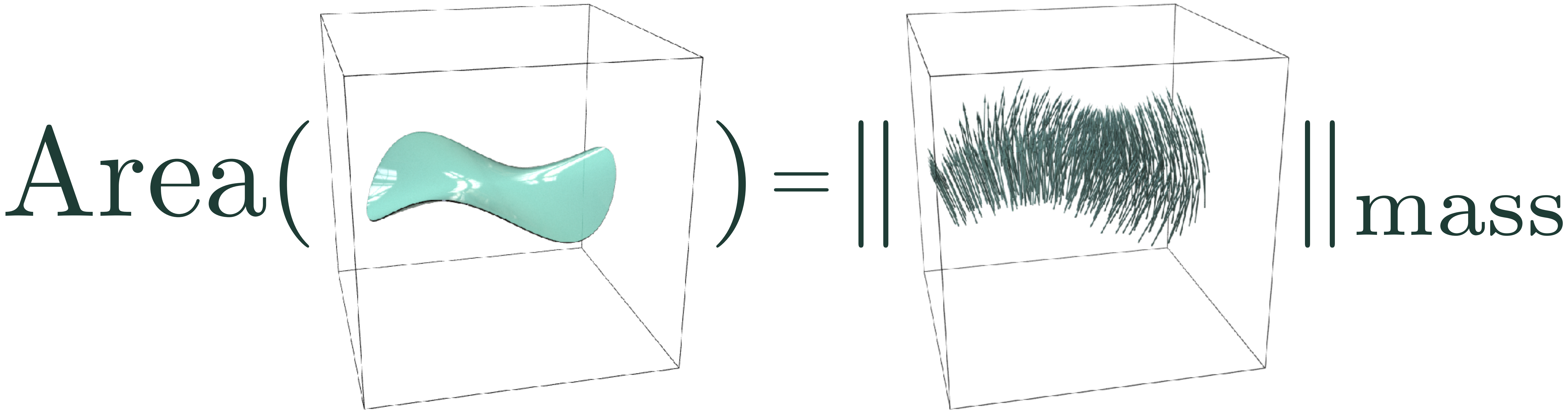
Plateau problem

(the problem of minimal surfaces)

Variable δ_Σ

Constraint $d\delta_\Sigma = \delta_\Gamma$ (a given space curve)

Objective $\|\delta_\Sigma\|_{\text{mass}}$



Plateau problem

(the problem of minimal surfaces)

Variable δ_Σ

Constraint $d\delta_\Sigma = \delta_\Gamma$ — a given space curve

Objective $\|\delta_\Sigma\|_{\text{mass}}$

Plateau problem

(the problem of minimal surfaces)

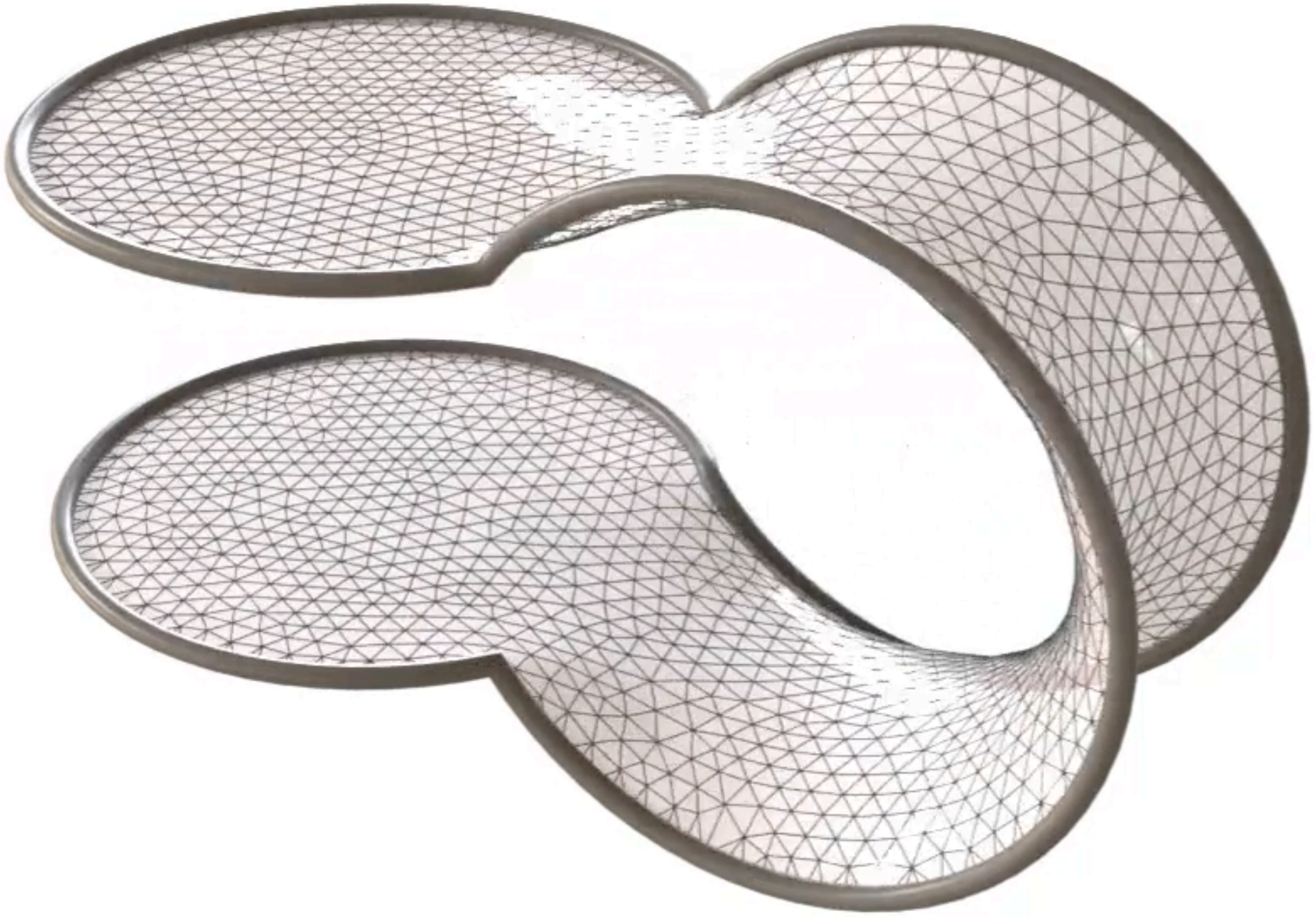
Variable	δ_Σ	
Constraint	$d\delta_\Sigma = \delta_\Gamma$	a given space curve
Objective	$\|\delta_\Sigma\|_{\text{mass}}$	

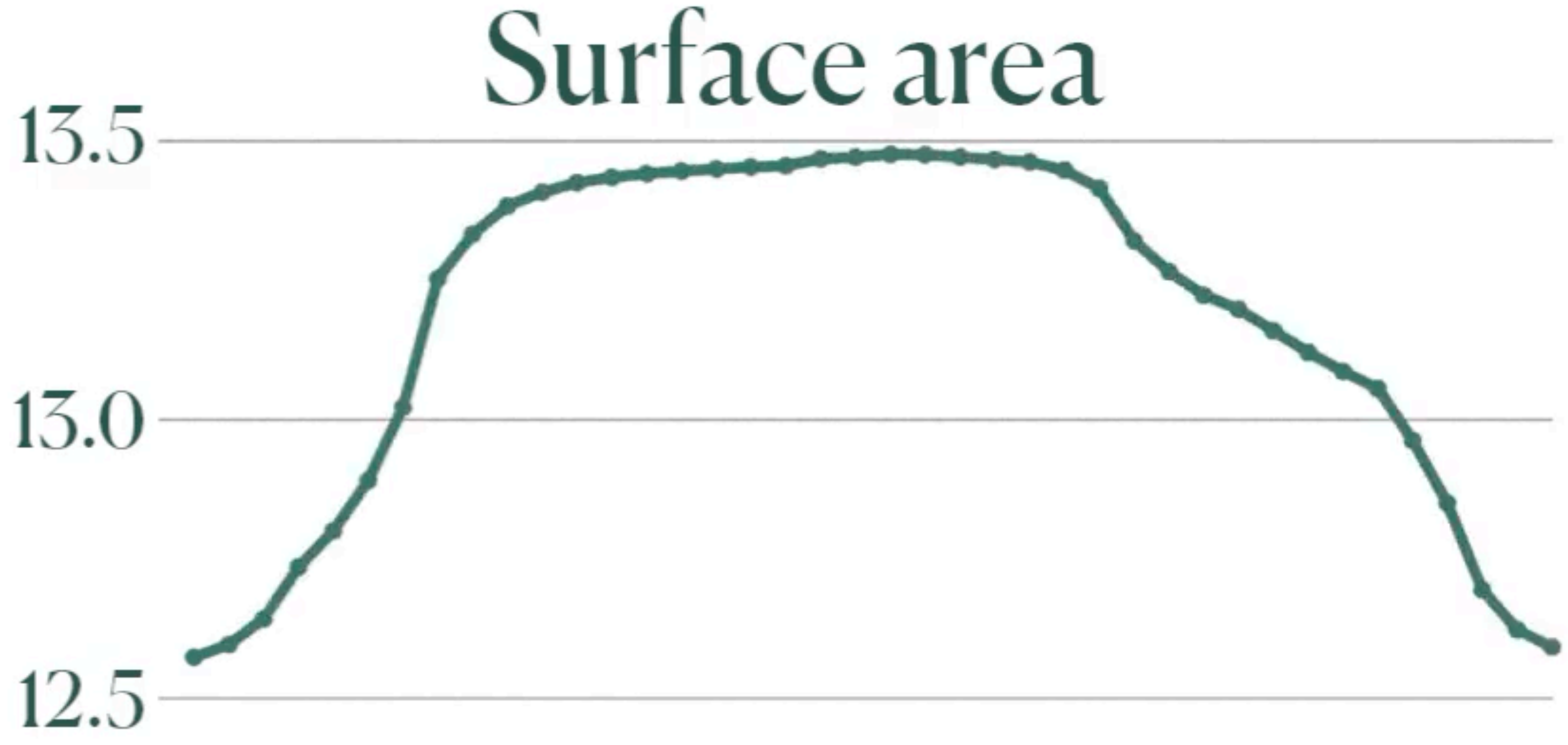
Variable	η	any 1-form
Constraint	$d\eta = \delta_\Gamma$	
Objective	$\|\eta\|_{\text{mass}}$	

a convex problem

“Area functional is nonconvex.”

—well-known geometric processing fact.





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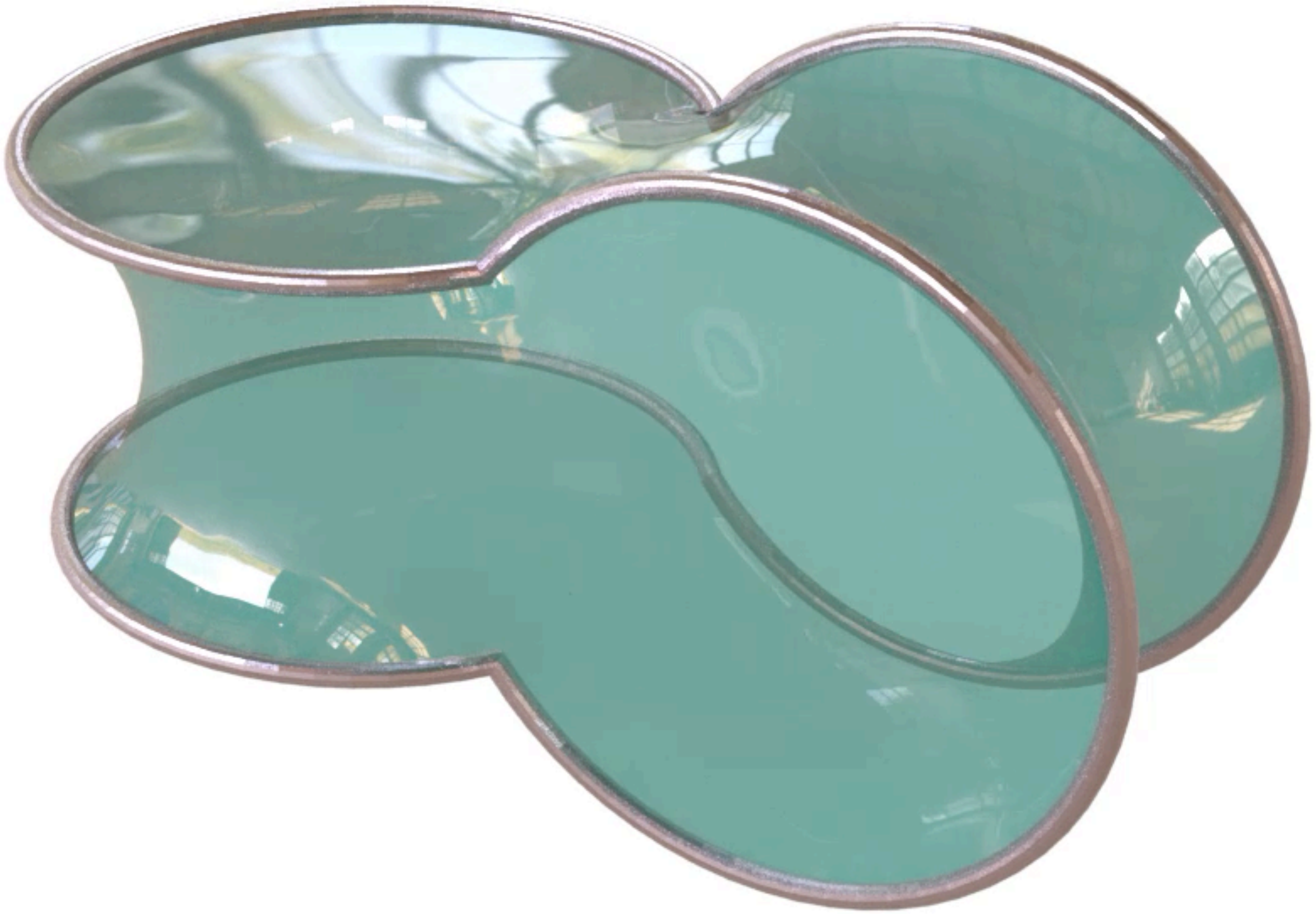
“Area functional is no longer nonconvex.”

—with surface represented with differential forms.



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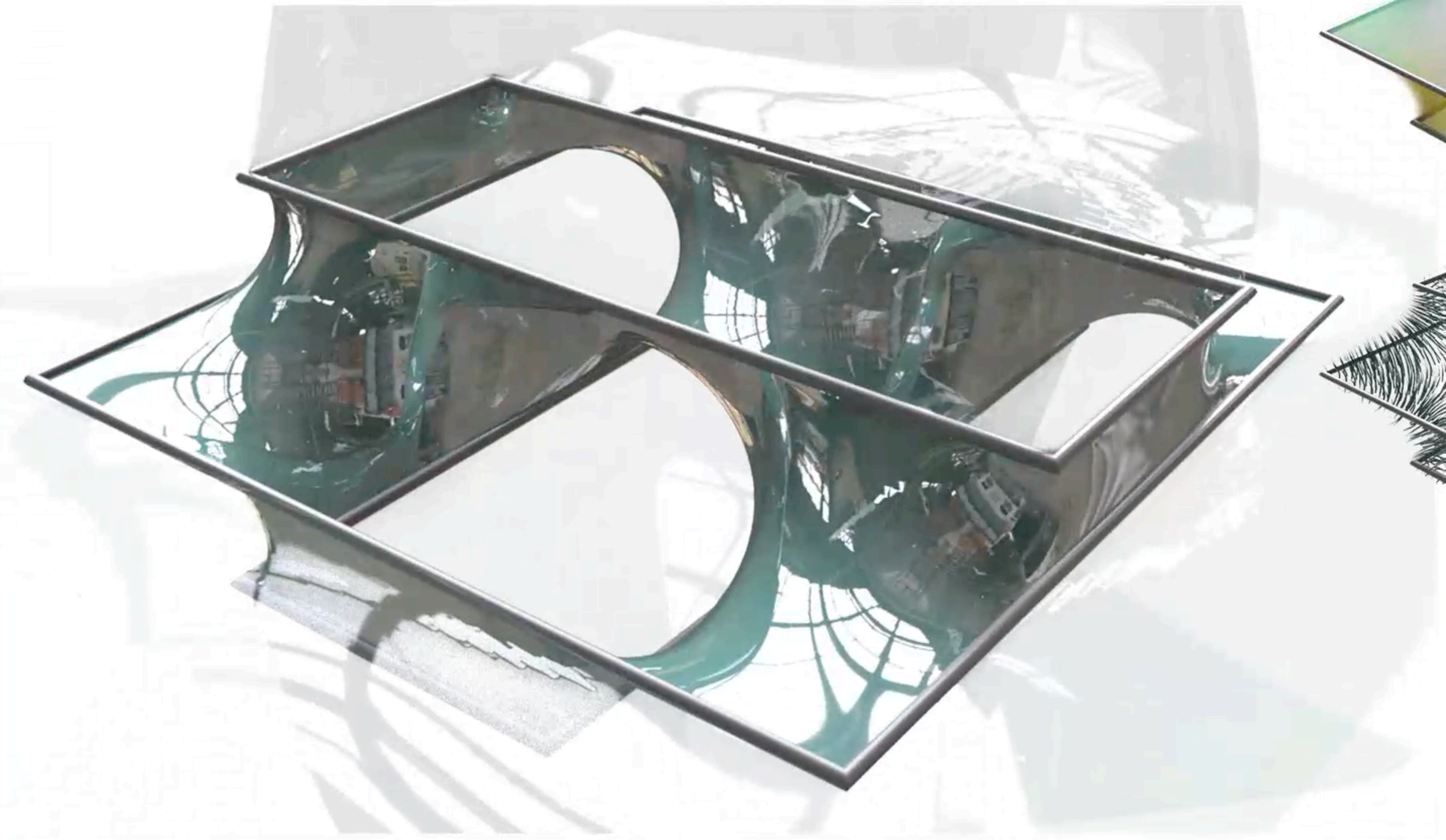

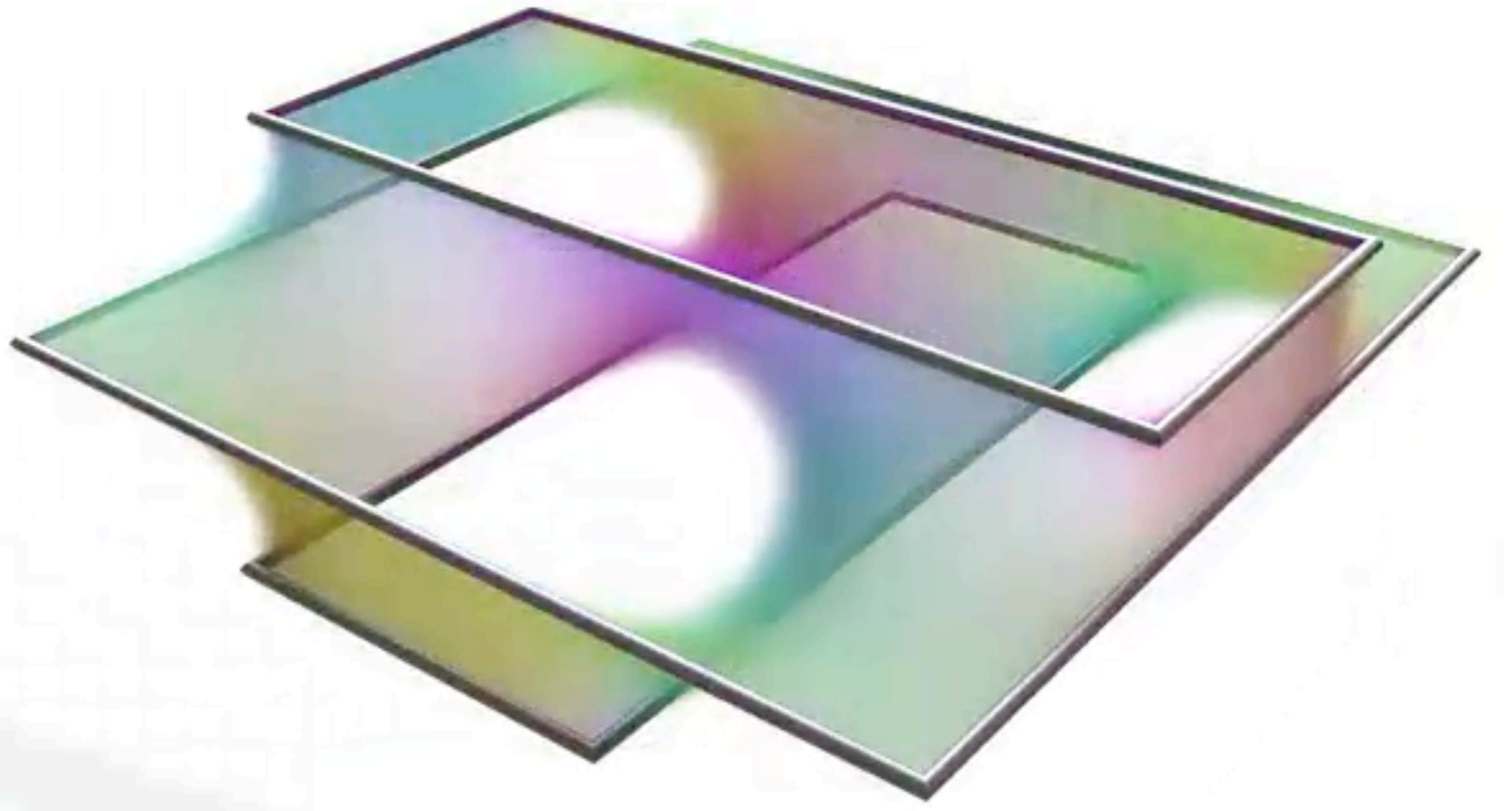

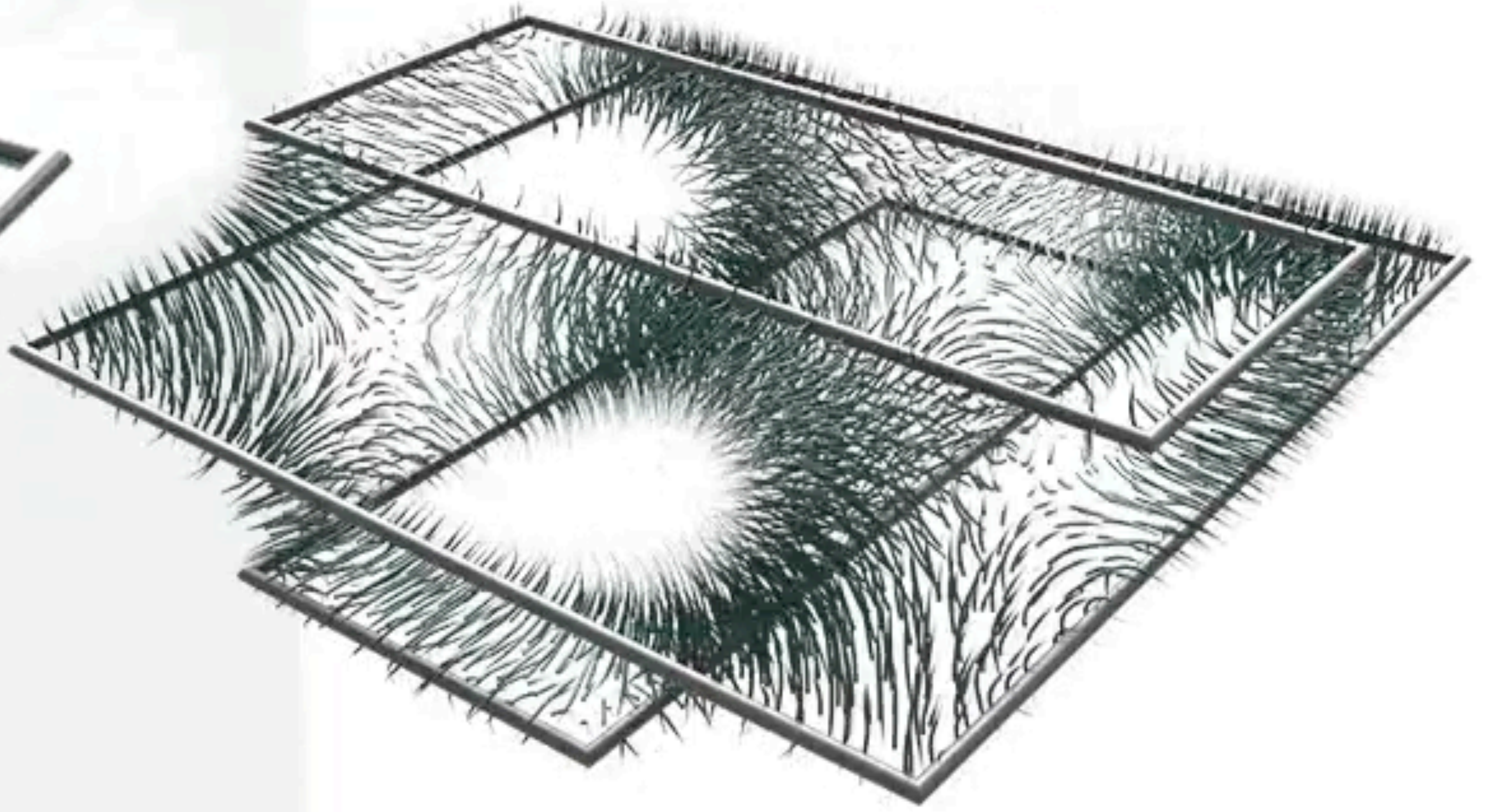

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Relaxed Plateau problem

Variable η any 1-form

Constraint $d\eta = \delta_\Gamma$ a given space curve

Objective $\|\eta\|_{\text{mass}}$

if $H^1(M) = 0$

Feasible set $\mathcal{A} = \{\eta : d\eta = \delta_\Gamma\} = \eta_0 + \ker(d^1) = \eta_0 + \text{im}(d^0)$

an arbitrary feasible solution

$= \{\eta_0 + d\varphi : \varphi \in \Omega^0(M)\}$

$\eta = \eta_0 + d\varphi$

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Fast ADMM algorithm

$$\underset{\varphi}{\text{minimize}}\ \langle \lambda, d\varphi \rangle_{L^2(M)} + \frac{\tau}{2}\| d\varphi - \hat{\eta} + \eta_0 \|^2_{L^2(M)}$$

complete squares

$$\Longleftrightarrow \quad \underset{\varphi}{\text{minimize}}\ \| d\varphi - \text{something} \|^2_{L^2(M)}$$

$$\Longleftrightarrow \quad \Delta\varphi = \text{some source term}$$

$$\underset{\eta}{\text{minimize}} \sum_p |\eta_p| - \langle \lambda_p, \eta_p \rangle + \frac{\tau}{2} |d\hat{\varphi}_p - \eta_p + \eta_{0,p}|^2$$

$$\underset{\mathbf{x}}{\text{minimize}}\ |\mathbf{x}| + \frac{\tau}{2}|\mathbf{x} - \text{something}|^2$$

$$\mathbf{x} = \text{Shrink}_{\frac{1}{\tau}}(\text{something})$$

optimality formula

$$\Longleftrightarrow \quad \eta_p = \text{Shrink}_{\frac{1}{\tau}}(\text{some pointwise attribute})$$

Relaxed Plateau problem

Variable η any 1-form

Constraint $d\eta = \delta_\Gamma$ a given space curve

Objective $\|\eta\|_{\text{mass}}$

if $H^1(M) = 0$

Feasible set $\mathcal{A} = \{\eta : d\eta = \delta_\Gamma\} = \eta_0 + \ker(d^1) = \eta_0 + \operatorname{im}(d^0)$

an arbitrary feasible solution

$= \{\eta_0 + d\varphi : \varphi \in \Omega^0(M)\}$

$$\eta = \eta_0 + d\varphi$$

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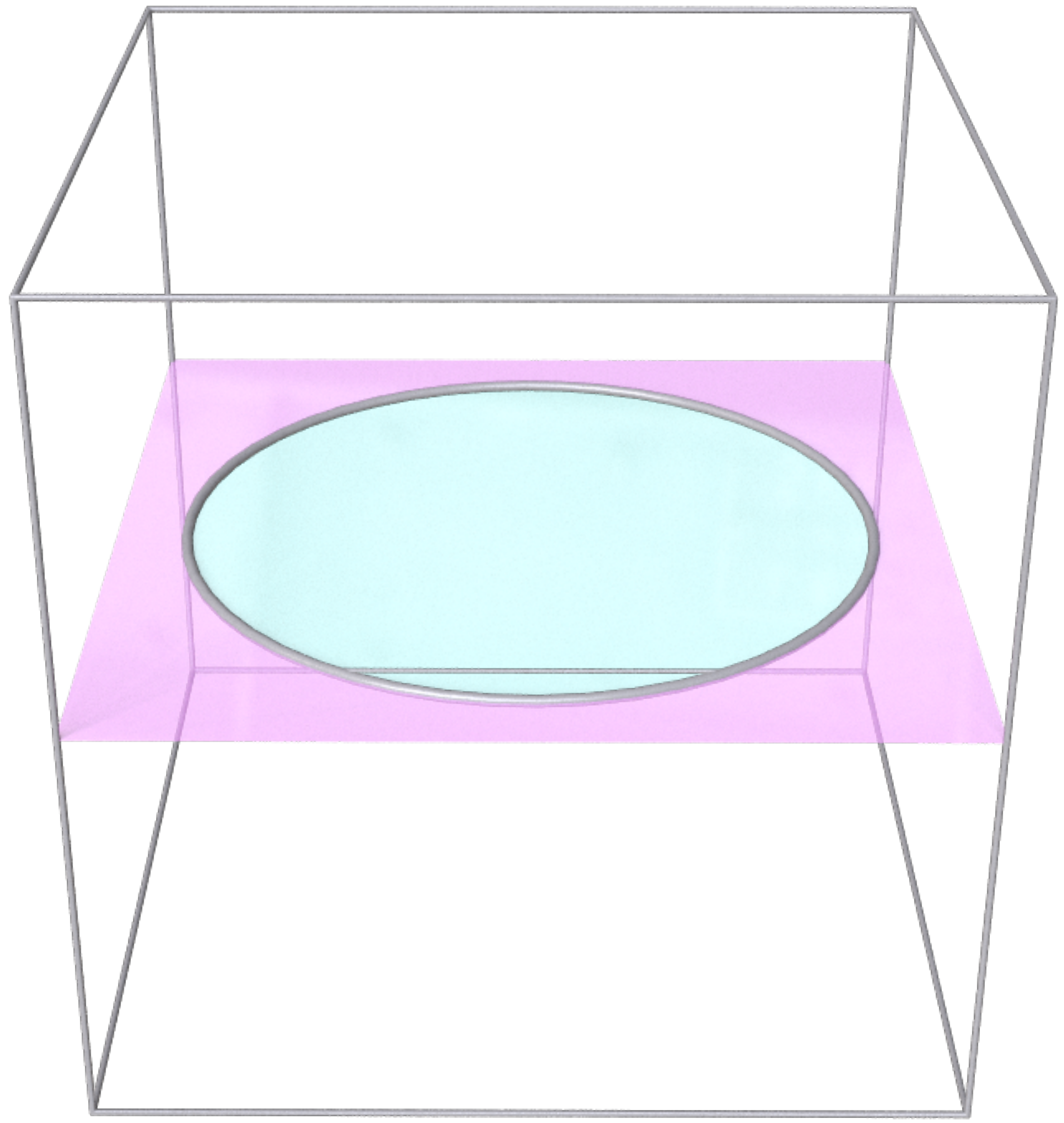
Plateau problem on 3-torus

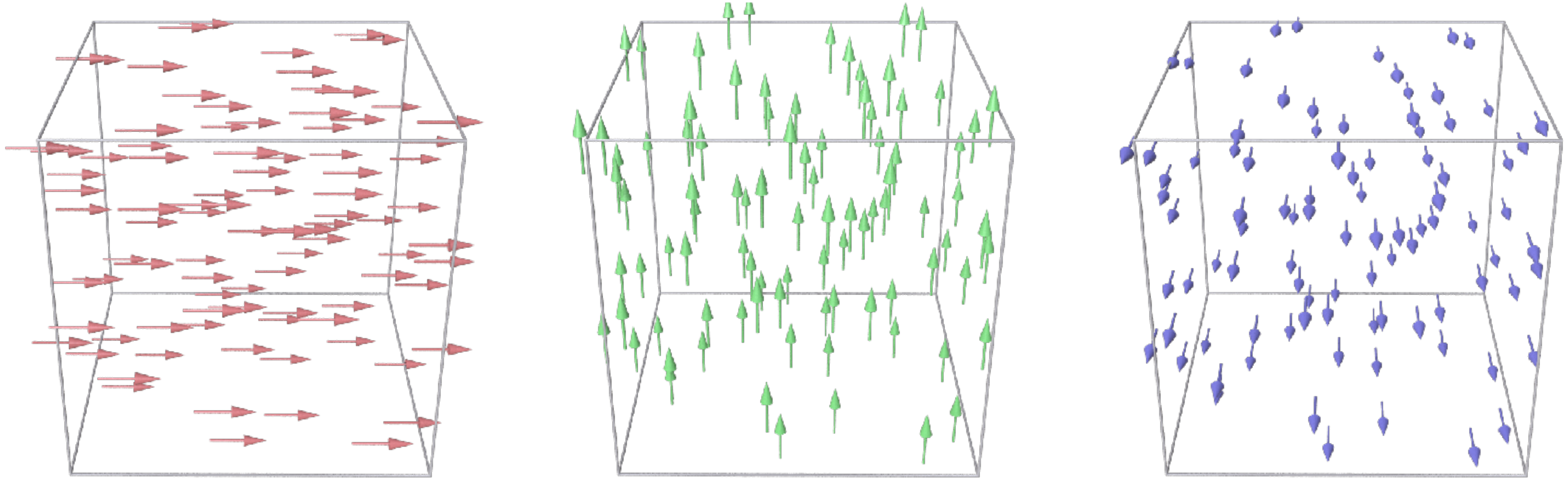
periodic boundary artifact



What happened on the 3-torus?

Cohomology matters

- Nontrivial cohomology $H^1(\mathbb{T}^3) = \ker(d^1)/\mathrm{im}(d^0) \neq 0$
- Nontrivial harmonic forms (closed but not exact)



h_1 h_2 h_3

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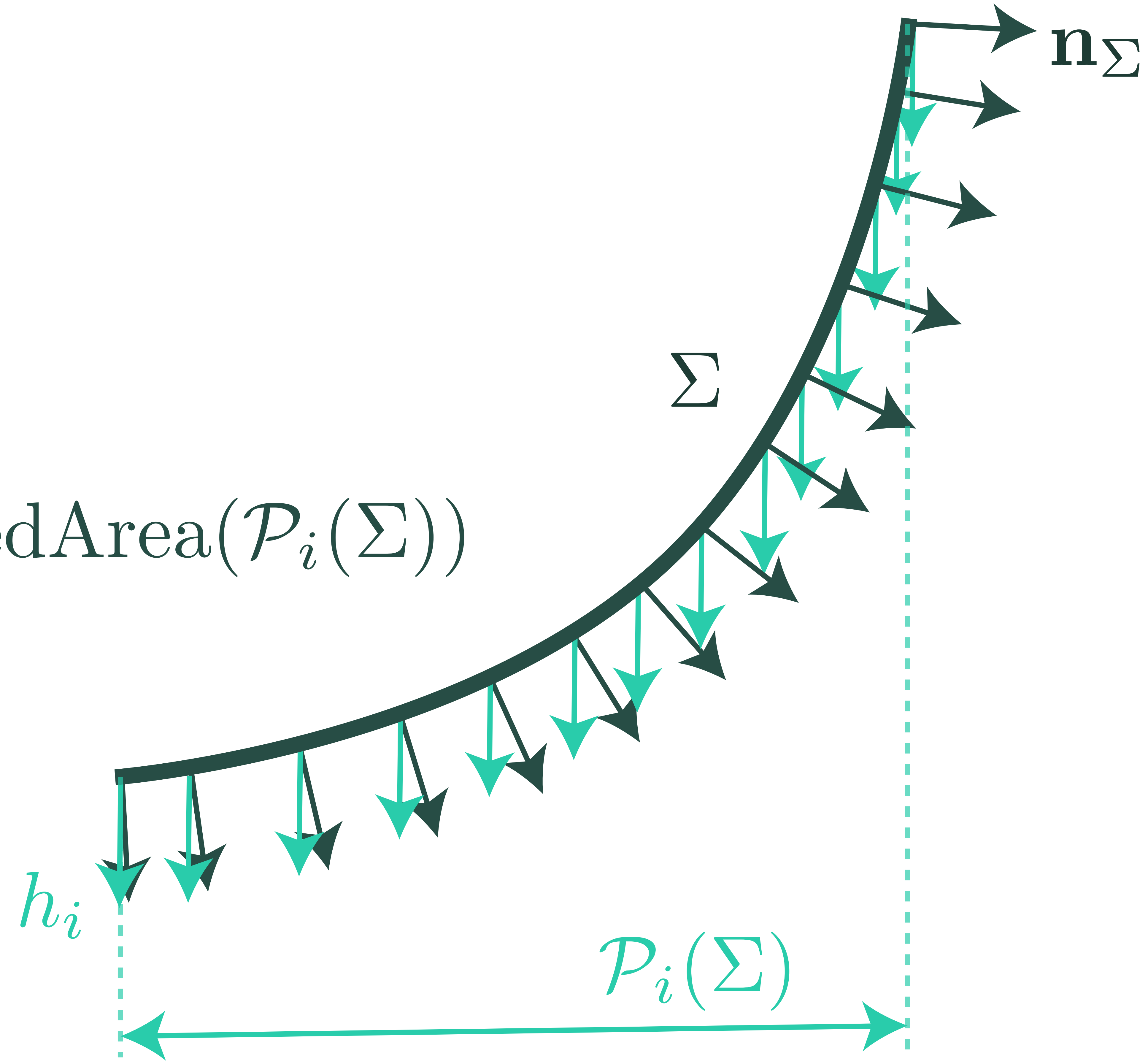
What happened on the 3-torus?

Cohomology matters

$$\int h_i \wedge \delta_\Sigma = \int_\Sigma \mathbf{e}_i \cdot \mathbf{n}_\Sigma dS$$

$$= \int_{\mathcal{P}_i(\Sigma)} 1 dS = \text{SignedArea}(\mathcal{P}_i(\Sigma))$$

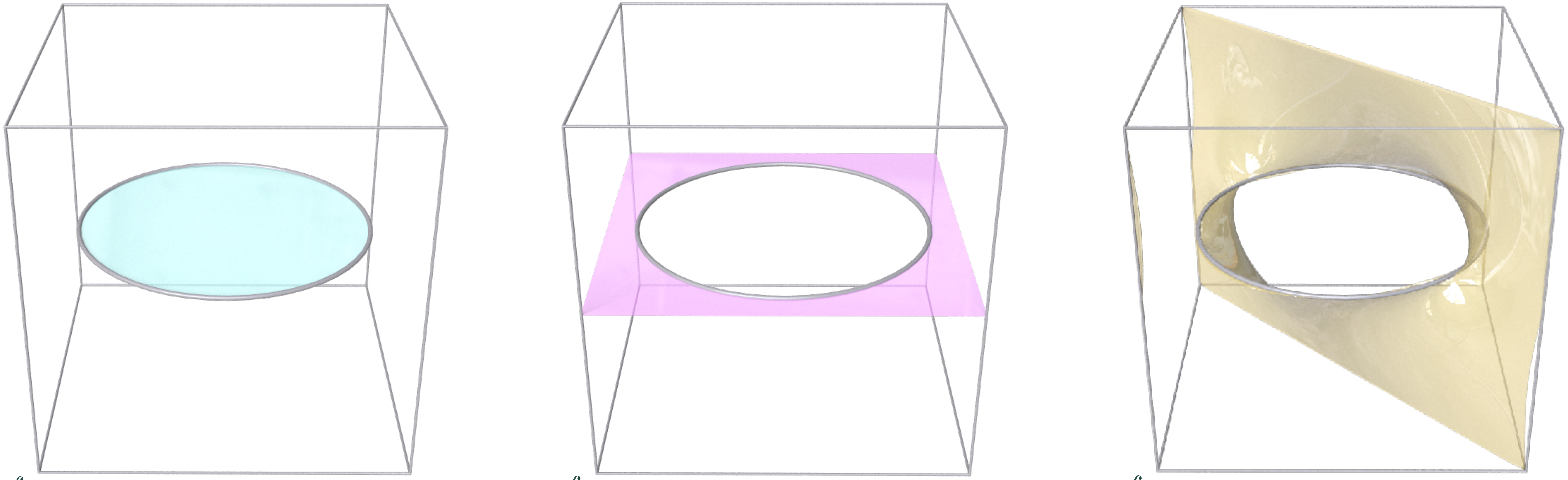


Cohomology constraint

$$\int h_i \wedge \delta_\Sigma = \int_\Sigma \mathbf{e}_i \cdot \mathbf{n}_\Sigma dS$$

$$= \int_{\mathcal{P}_i(\Sigma)} 1 dS = \mathrm{SignedArea}(\mathcal{P}_i(\Sigma))$$

examples



$$\int h_1 \wedge \eta = 0$$

$$\int h_2 \wedge \eta = \mathrm{Area}(\mathbb{D})$$

$$\int h_3 \wedge \eta = 0$$

$$\int h_1 \wedge \eta = 0$$

$$\int h_2 \wedge \eta = 1 - \mathrm{Area}(\mathbb{D})$$

$$\int h_3 \wedge \eta = 0$$

$$\int h_1 \wedge \eta = -1$$

$$\int h_2 \wedge \eta = 1 - \mathrm{Area}(\mathbb{D})$$

$$\int h_3 \wedge \eta = 1$$

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Admissible set

Cohomology constraint

- Adding Cohomology constraint

$$\int h_i \wedge \eta = \psi_i, i = 1, 2, 3$$

- Admissible set

$$\mathcal{A} = \left\{ \eta \in \mathcal{D}\Omega^1(M) \middle| d\eta = \delta_\Gamma, \int h_i \wedge \eta = \psi_i, i = 1, 2, 3 \right\}$$

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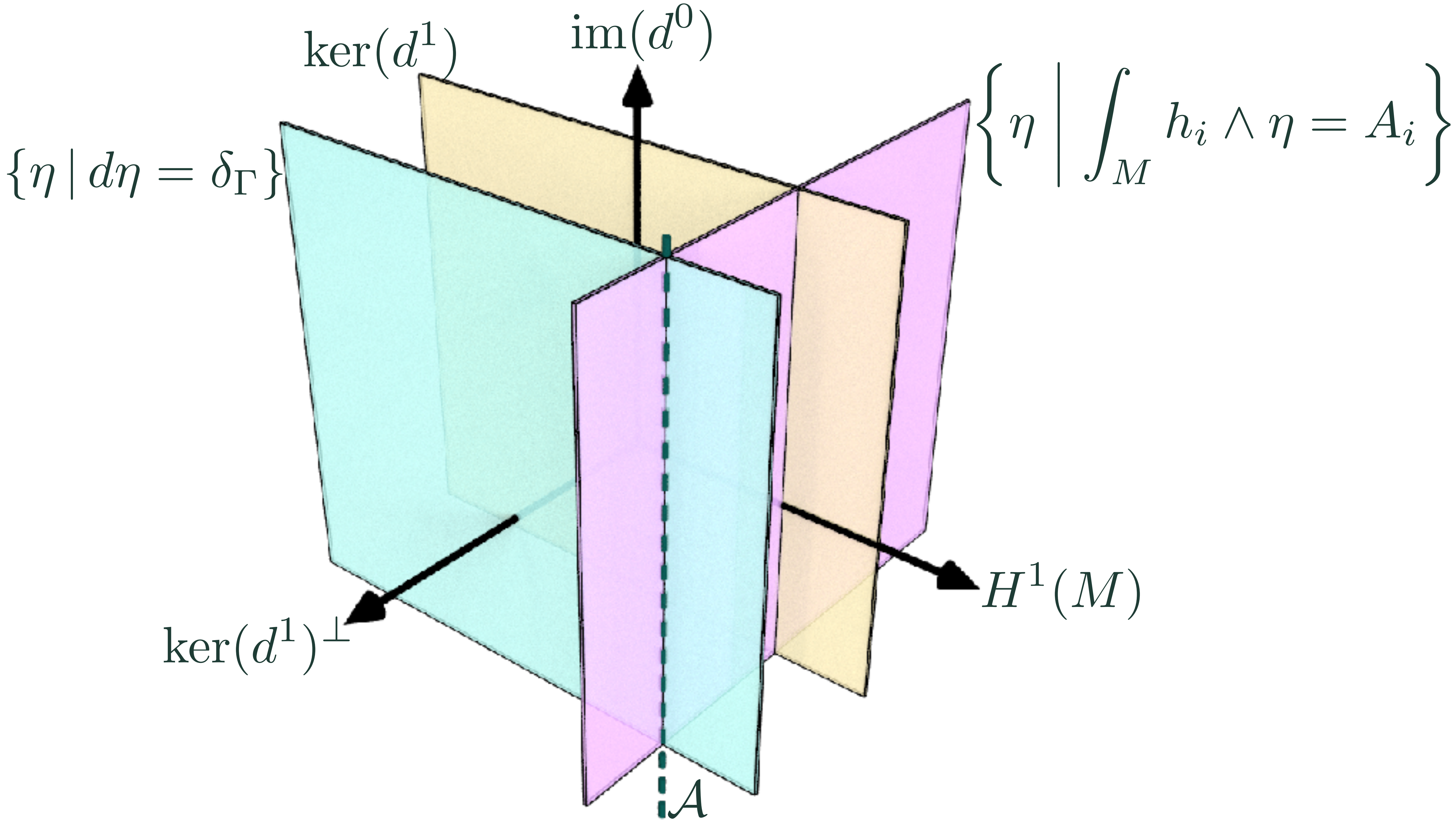
Admissible set

Cohomology constraint

$$\mathcal{A} = \left\{ \eta \in \mathcal{D}\Omega^1(M) \,\middle|\, d\eta = \delta_\Gamma, \int h_i \wedge \eta = \psi_i, i = 1, 2, 3 \right\}$$



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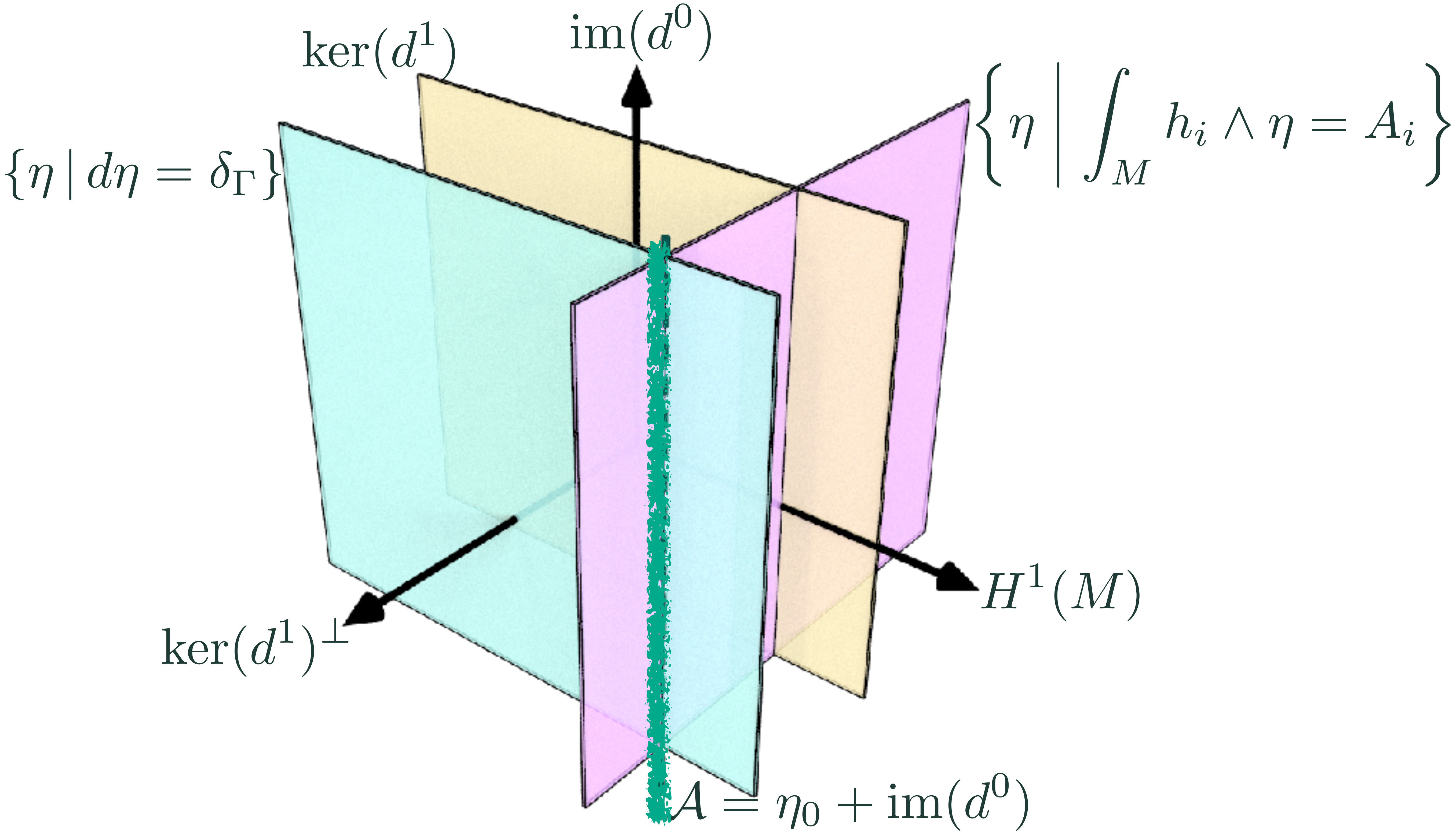
Admissible set

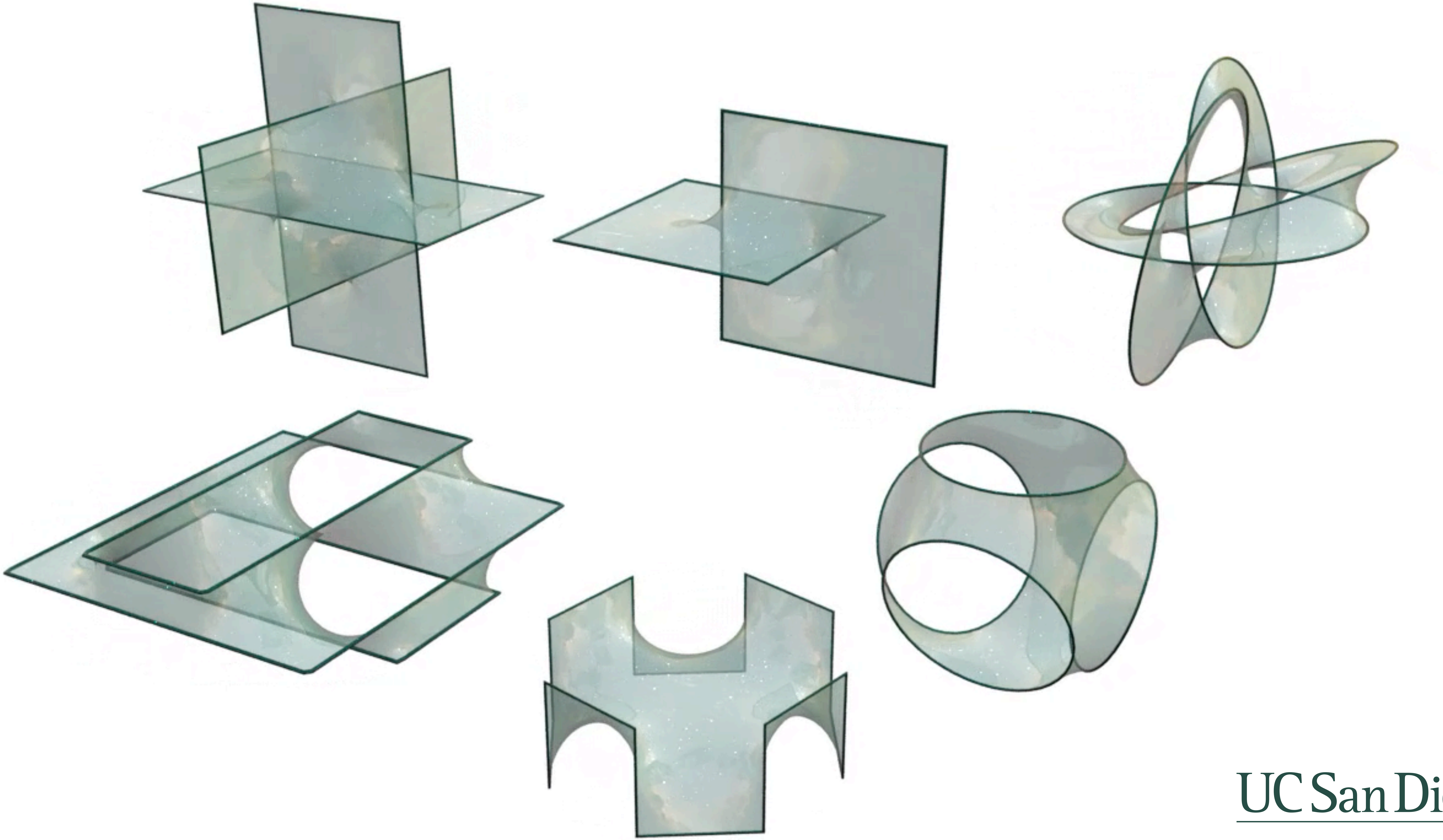
Cohomology constraint

same as simply connected ambient space!

$$\mathcal{A} = \left\{ \eta \in \mathcal{D}\Omega^1(M) \,\middle|\, d\eta = \delta_\Gamma, \int h_i \wedge \eta = \psi_i, i = 1, 2, 3 \right\} = \eta_0 + \mathrm{im}\left(d^0\right)$$



Results



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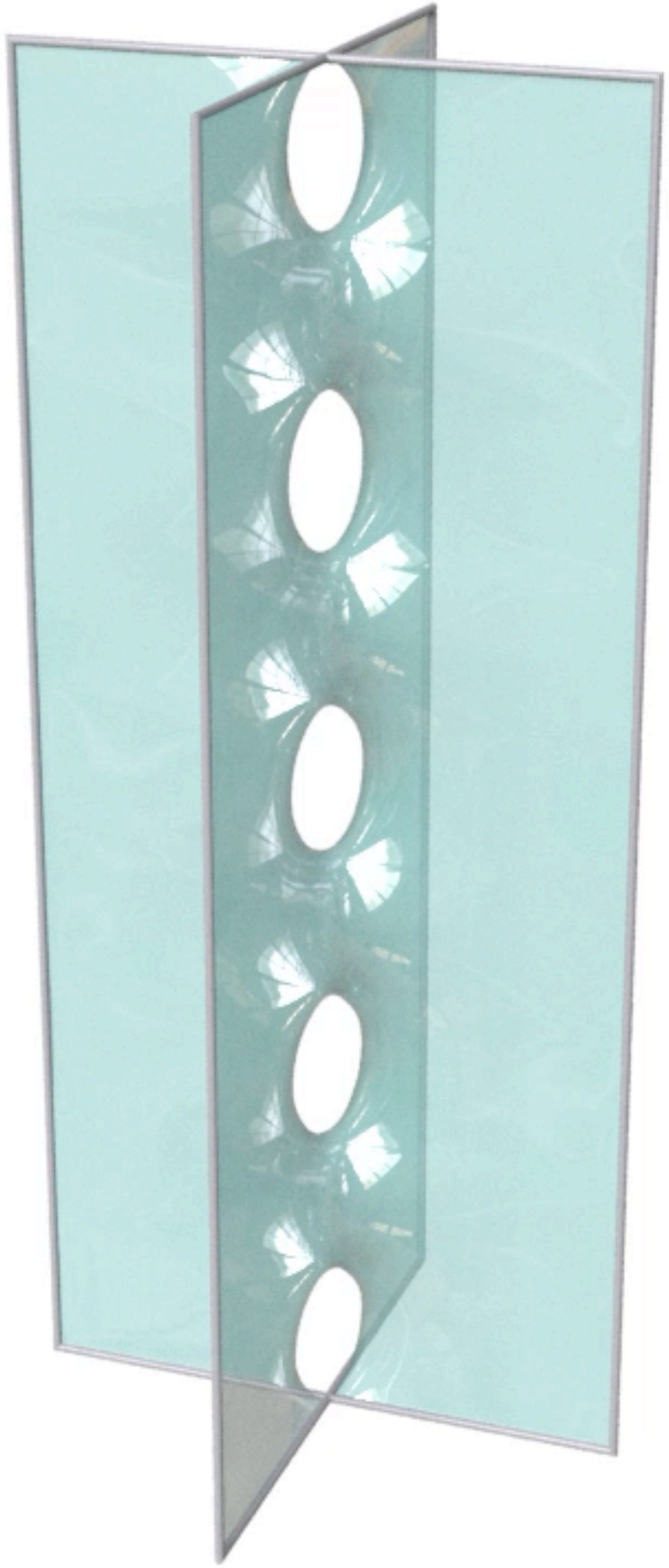
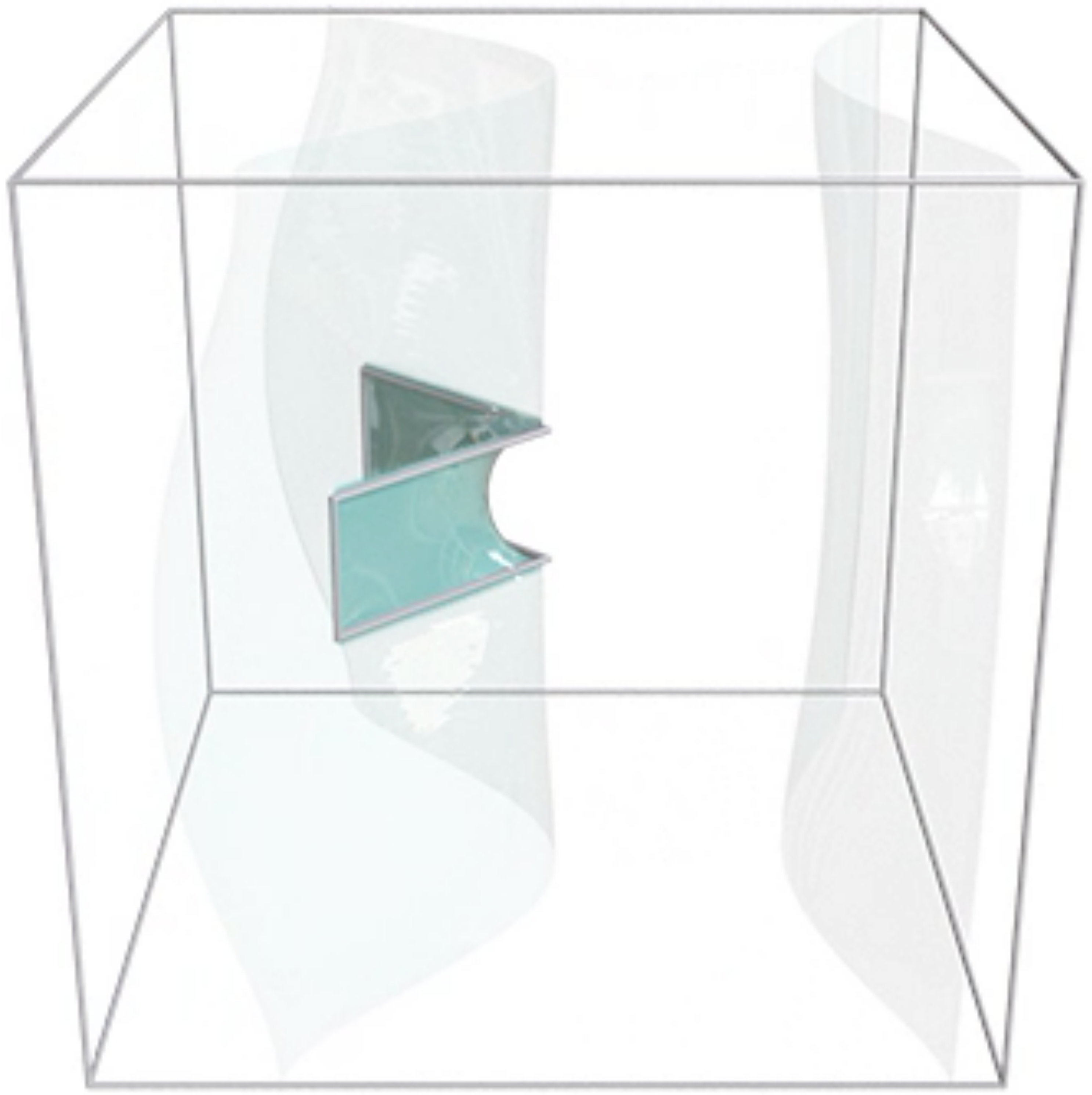
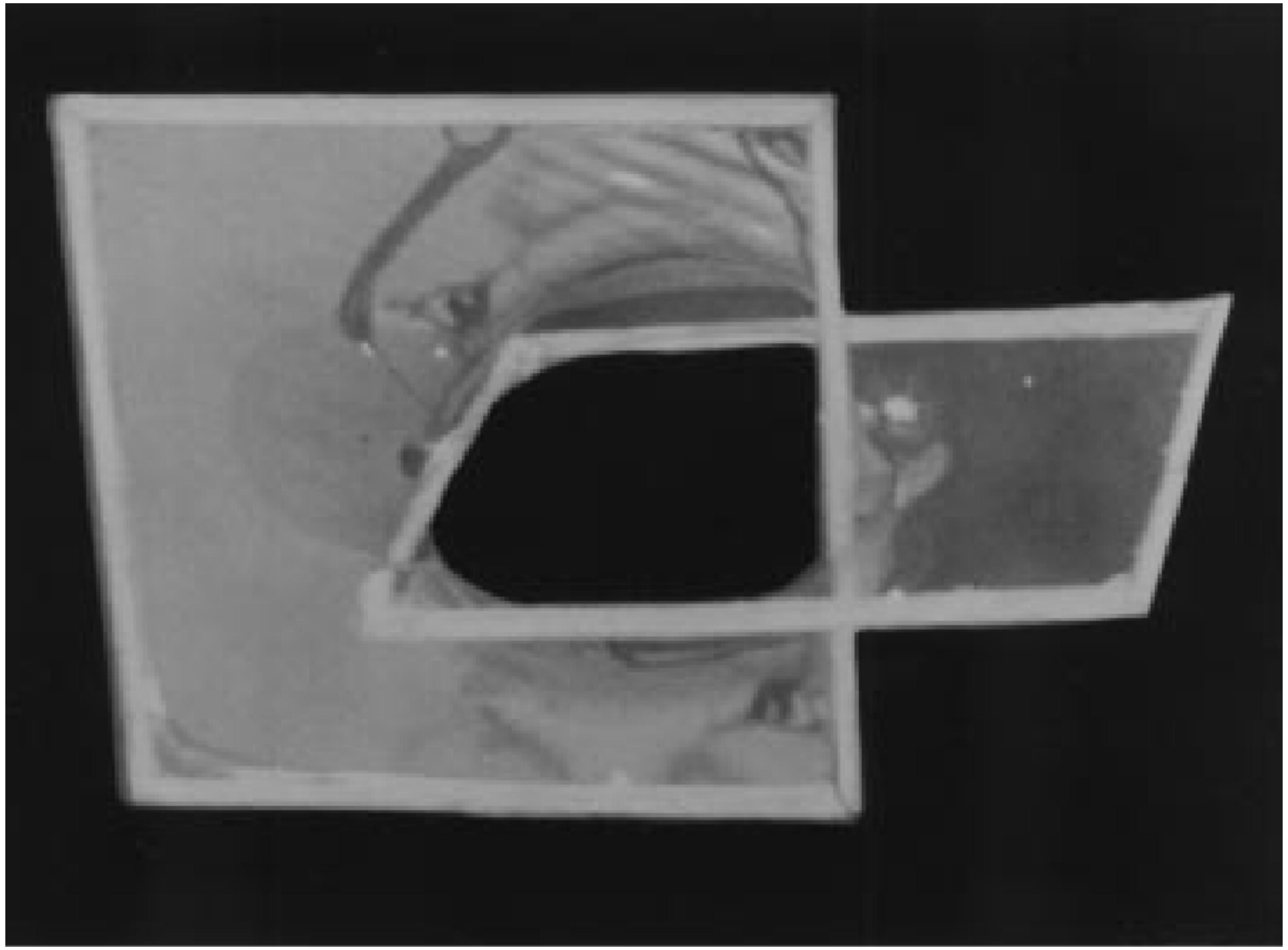
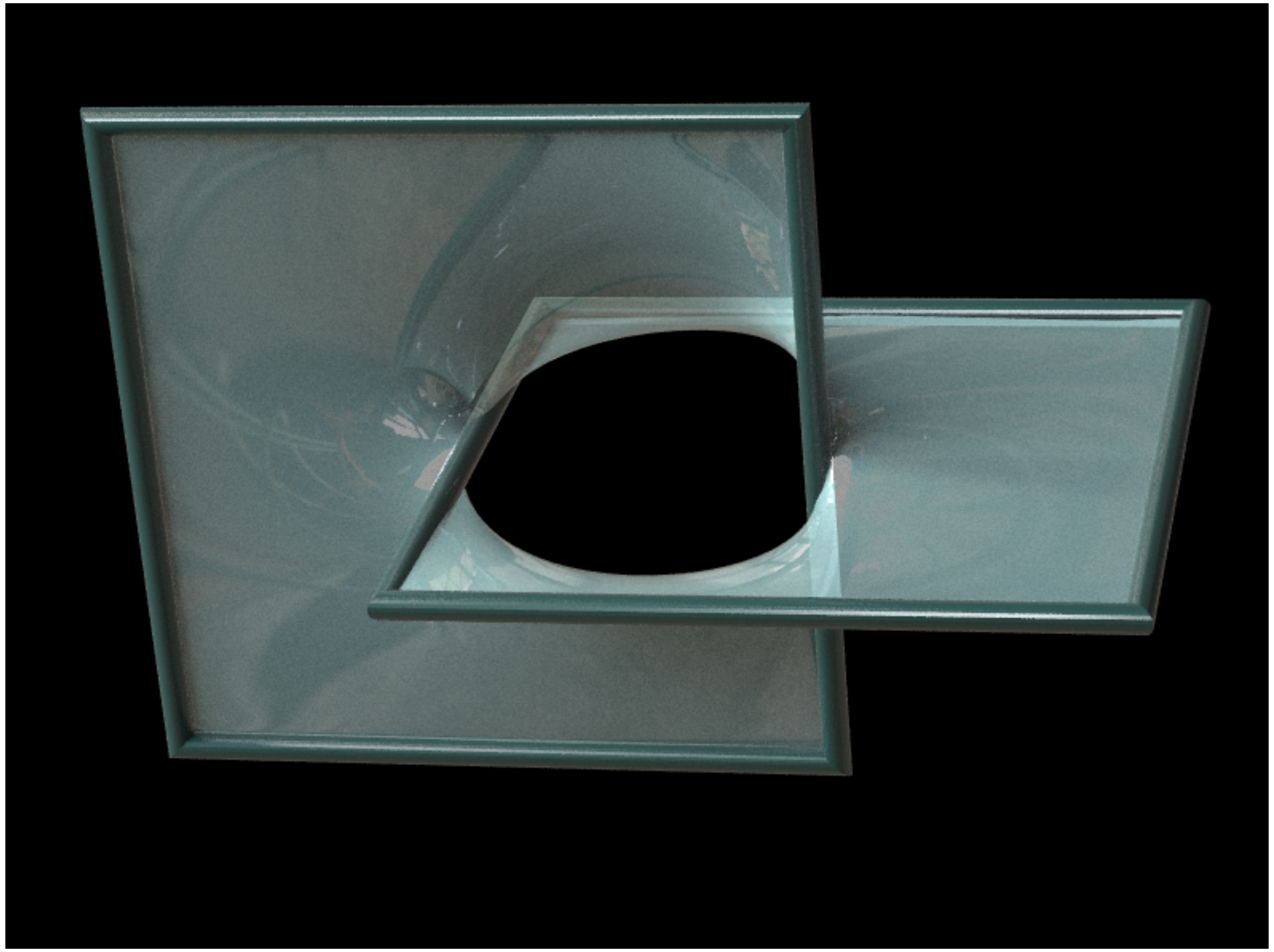




Comparison





PC: H. Parks and J. Pitts 1997

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Minimal surface

$$\underset{\eta : d\eta = \delta_\Gamma}{\text{minimize}} \ \|\eta\|_{\text{mass}}$$

Γ: input boundary curve

Geometric least-square problem

$$\underset{\eta : d\eta = \omega}{\text{minimize}} \ \|\eta\|_{L^2}$$

ω: some input differential form

Poisson surface reconstruction

$$\underset{\varphi}{\text{minimize}} \ \|\delta_\Sigma - d\varphi\|_{L^2}$$

Σ: input incomplete surface

Electricity field

$$\underset{\omega : d\omega = \rho}{\text{minimize}} \ \|\omega\|_{L^2}$$

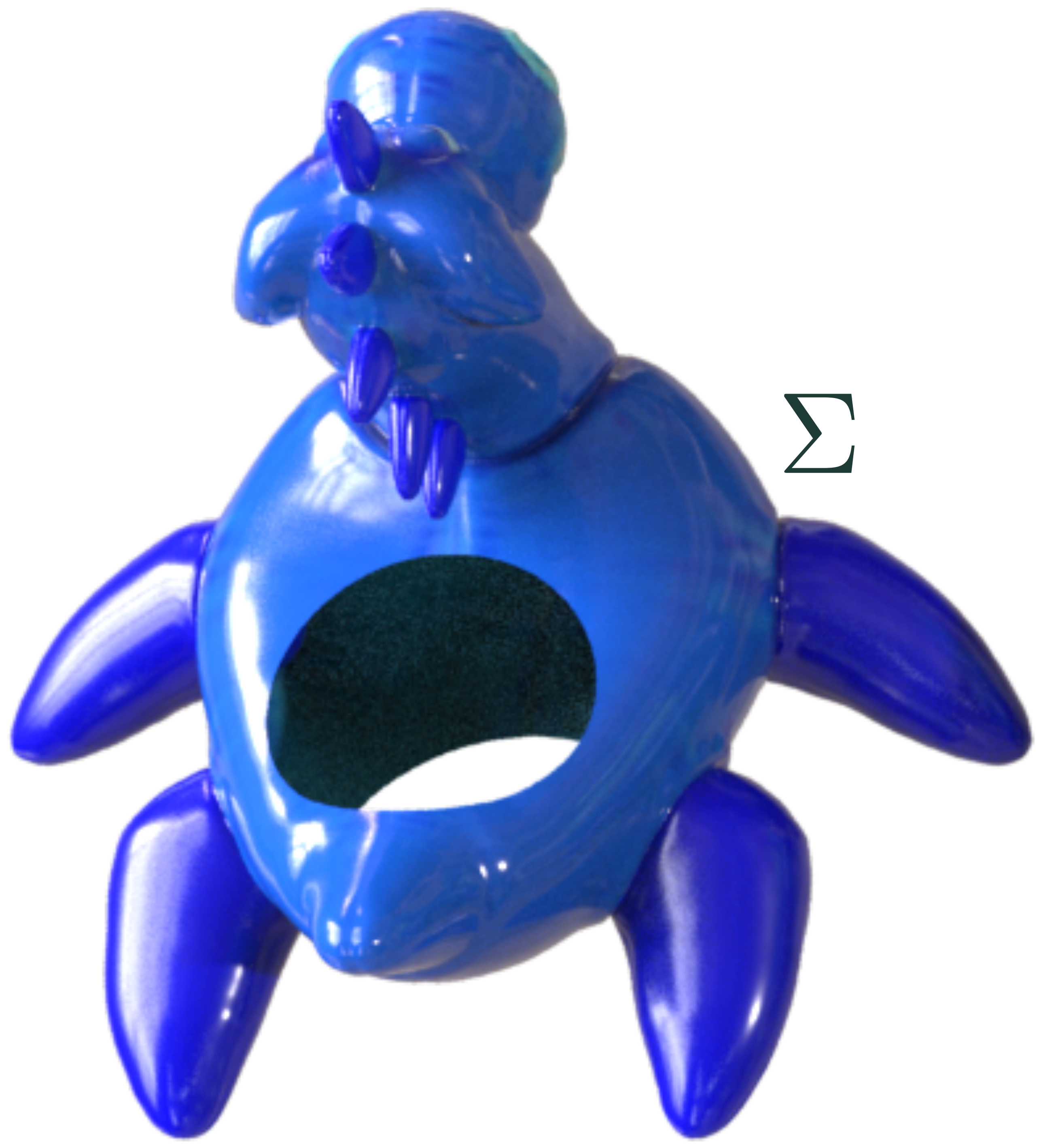
ρ: distribution of charged particles

General mass norm minimization

- Poisson surface reconstruction [Kazhdan et al. 2006]

$$\underset{\varphi}{\text{minimize}} \ \|\delta_\Sigma - d\varphi\|_{L^2}$$

- Replacing L2 norm with mass norm...



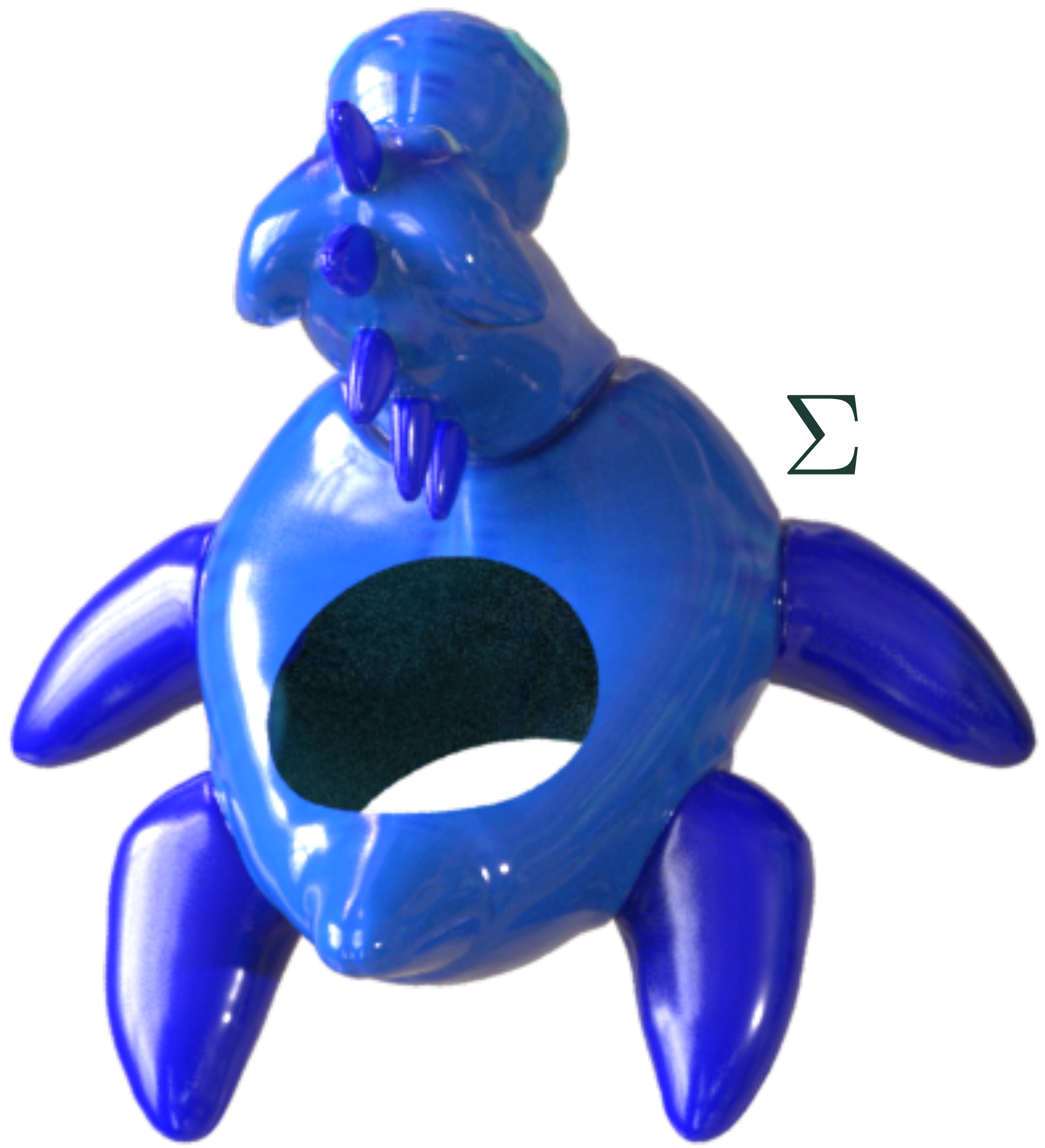
General mass norm minimization

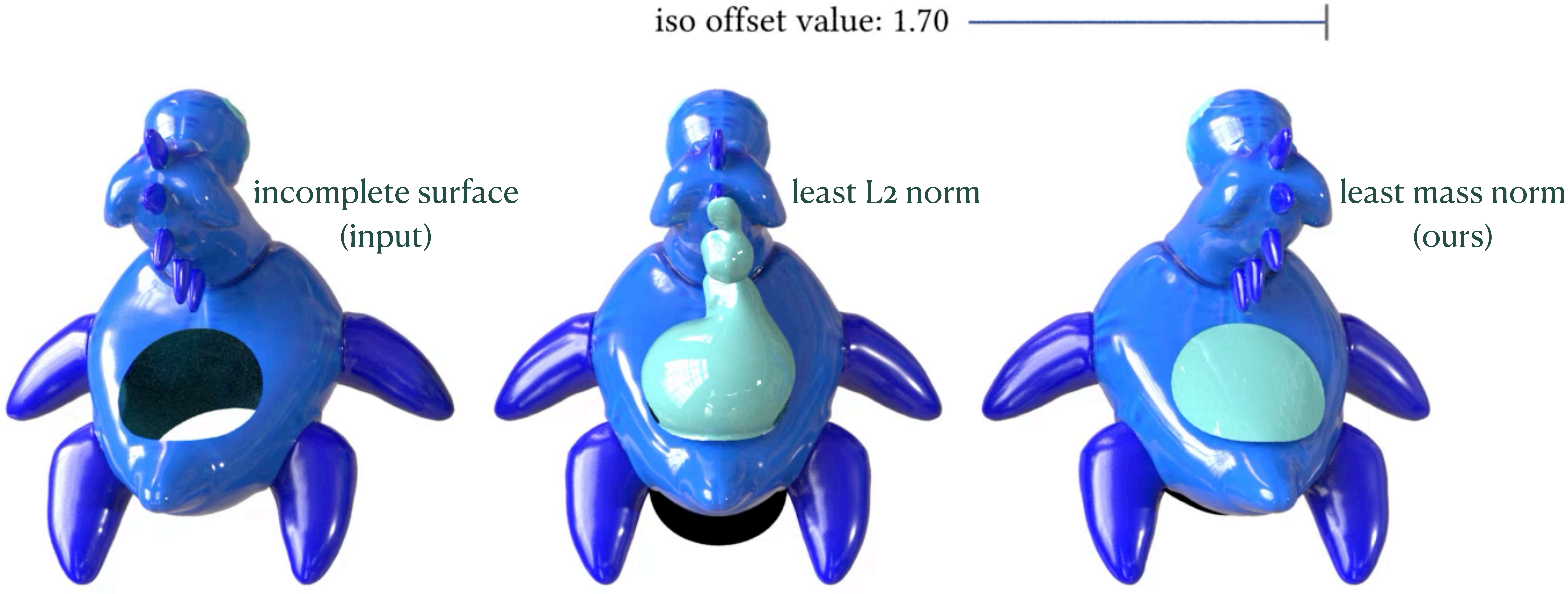
- Poisson surface reconstruction [Kazhdan et al. 2006]

$$\underset{\varphi}{\text{minimize}} \; \|\delta_\Sigma - d\varphi\|_{L^2}$$

- Replacing L2 norm with mass norm...

$$\underset{\varphi}{\text{minimize}} \; \|\delta_\Sigma - d\varphi\|_{\text{mass}}$$



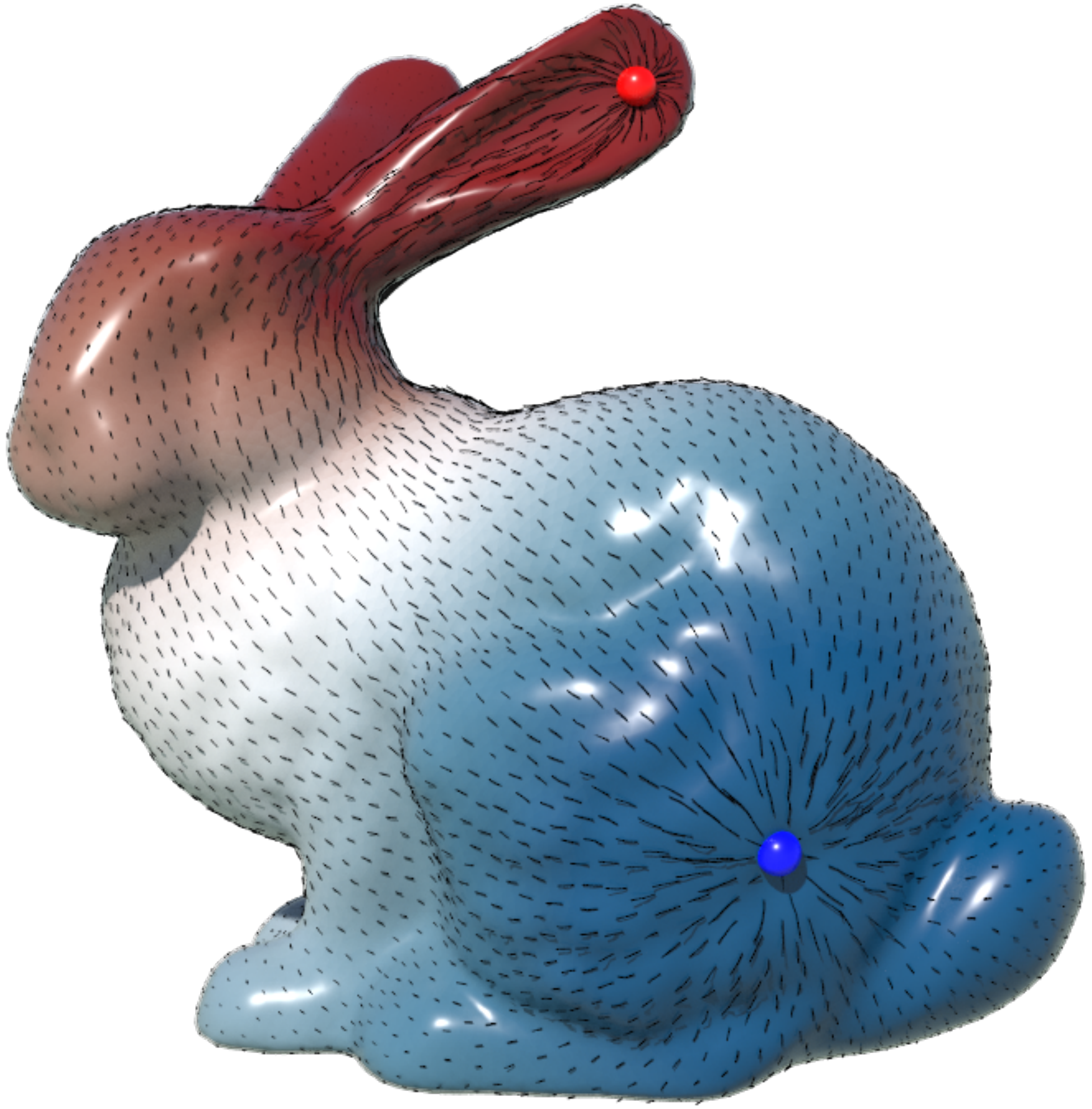

iso offset value: 1.70
incomplete surface
(input)
least L2 norm
least mass norm
(ours)

General mass norm minimization

- Electric field around charged particles

$$\underset{\omega : d\omega = \rho}{\text{minimize}} \quad \|\omega\|_{L^2}$$

- Replacing L2 norm with mass norm…



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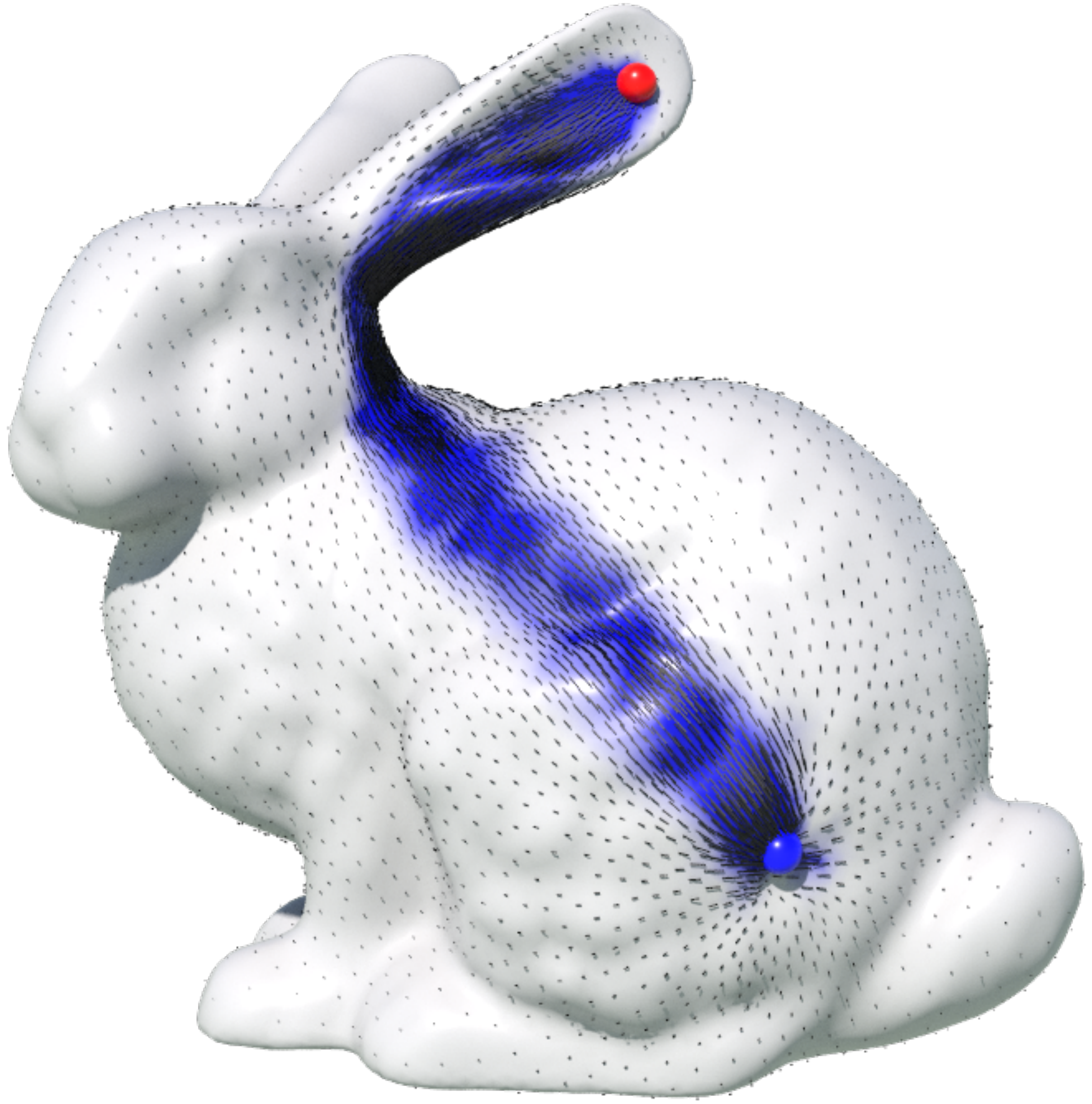
General mass norm minimization

- Electric field around charged particles

$$\underset{\omega : d\omega = \rho}{\text{minimize}} \; \|\omega\|_{L^2}$$

- Replacing L2 norm with mass norm...

$$\underset{\omega : d\omega = \rho}{\text{minimize}} \; \|\omega\|_{\text{mass}}$$



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General mass norm minimization

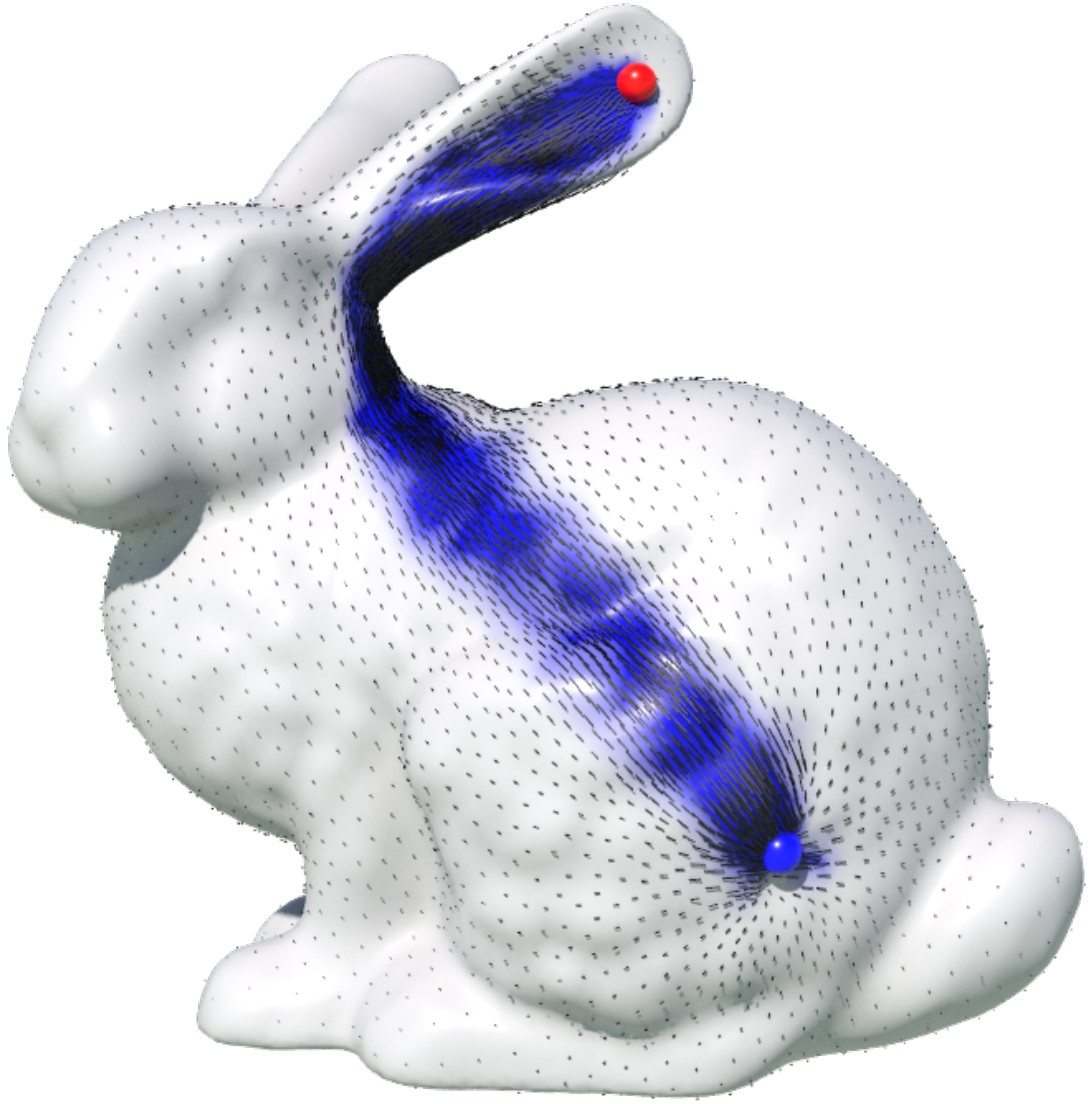
- Electric field around charged particles

$$\underset{\omega: d\omega=\rho}{\text{minimize}} \ \|\omega\|_{L^2}$$

- Replacing L2 norm with mass norm...

$$\underset{\omega: d\omega=\rho}{\text{minimize}} \ \|\omega\|_{\text{mass}}$$

$$\underset{\omega: d\omega=\rho_1-\rho_2}{\text{minimize}} \ \|\omega\|_{L^1}$$



Keywords: Beckmann problem, earth mover distance [Solomon et al. 2014].

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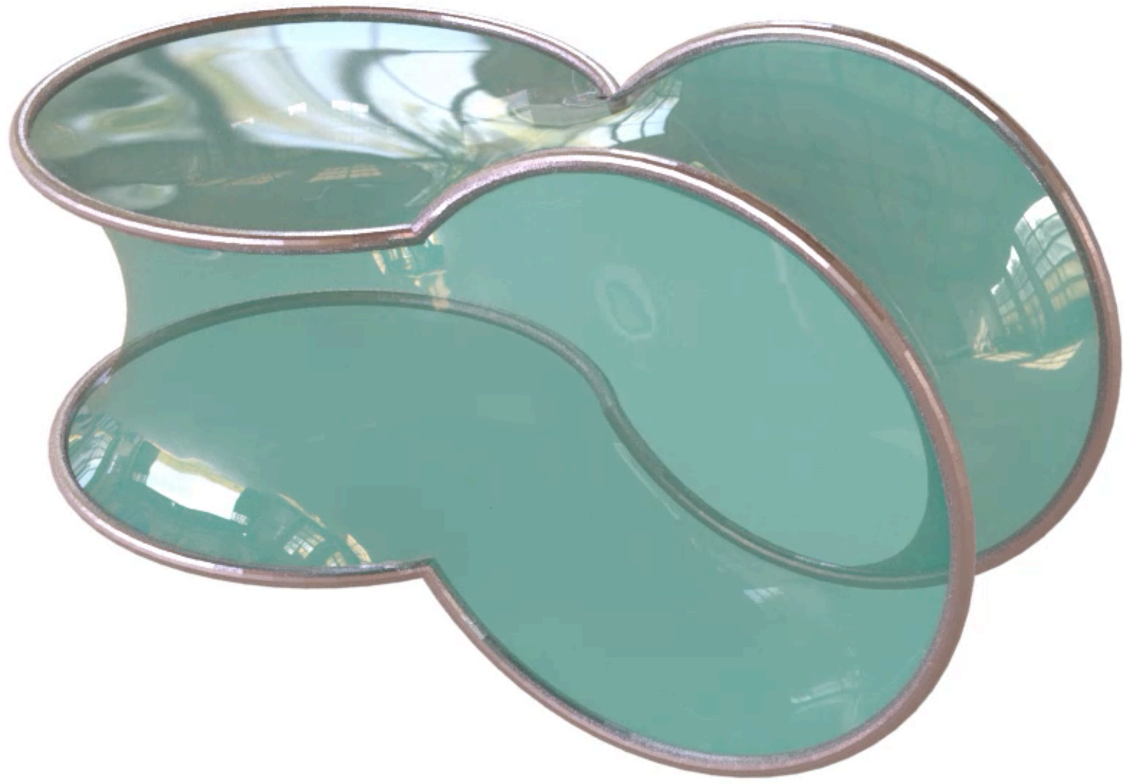
Summary

Eulerian geometry representation + general mass norm minimization

- **Differentiable** geometry representation for inverse rendering and 3D deep learning
- **Convex** representation via manifold superposition
- **Sparsity** provided by mass norm minimization



- Limitations: non-manifold surfaces, non-oriented surfaces

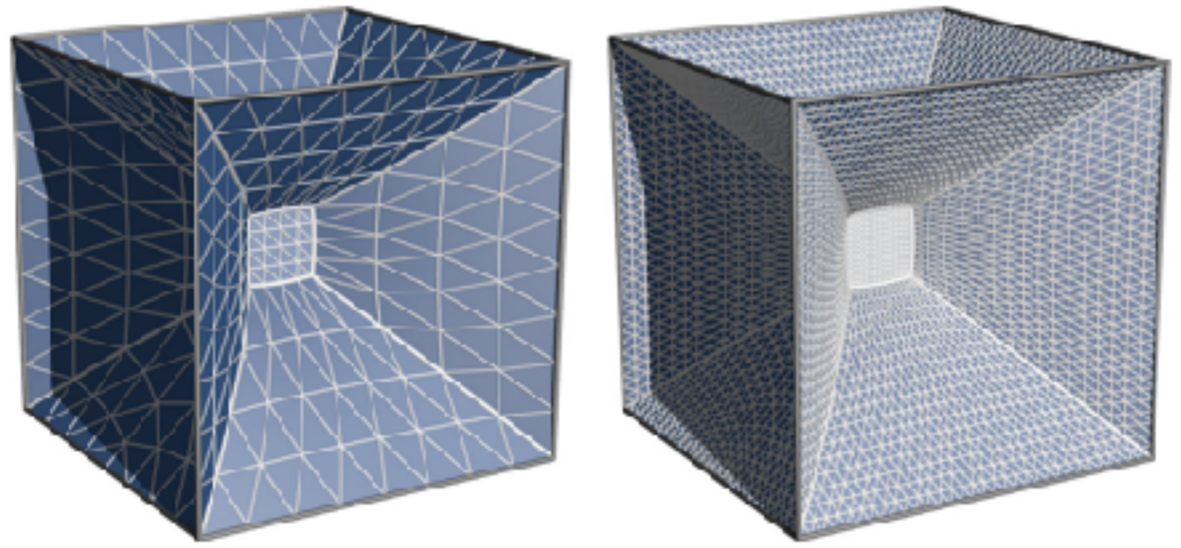


Figure courtesy: H. Schumacher and M. Wardetzky 2019

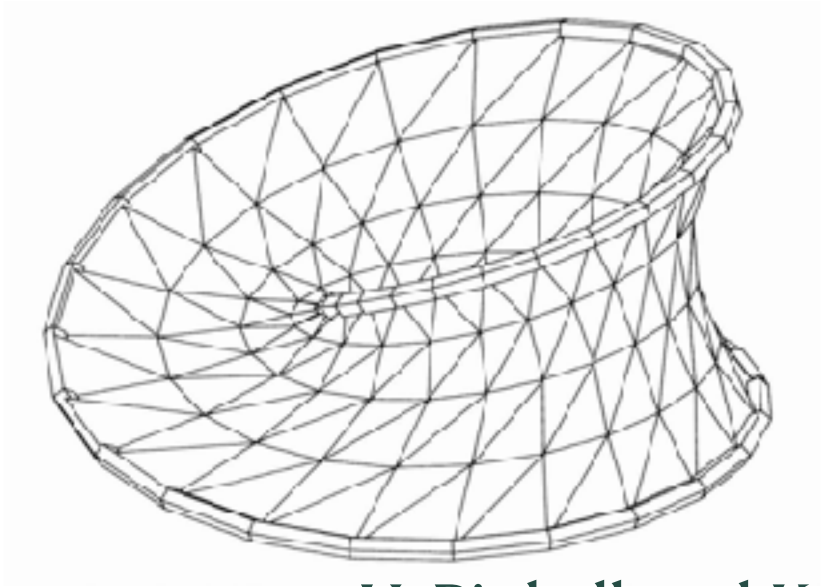


Figure courtesy: U. Pinkall and K. Polthier 1993

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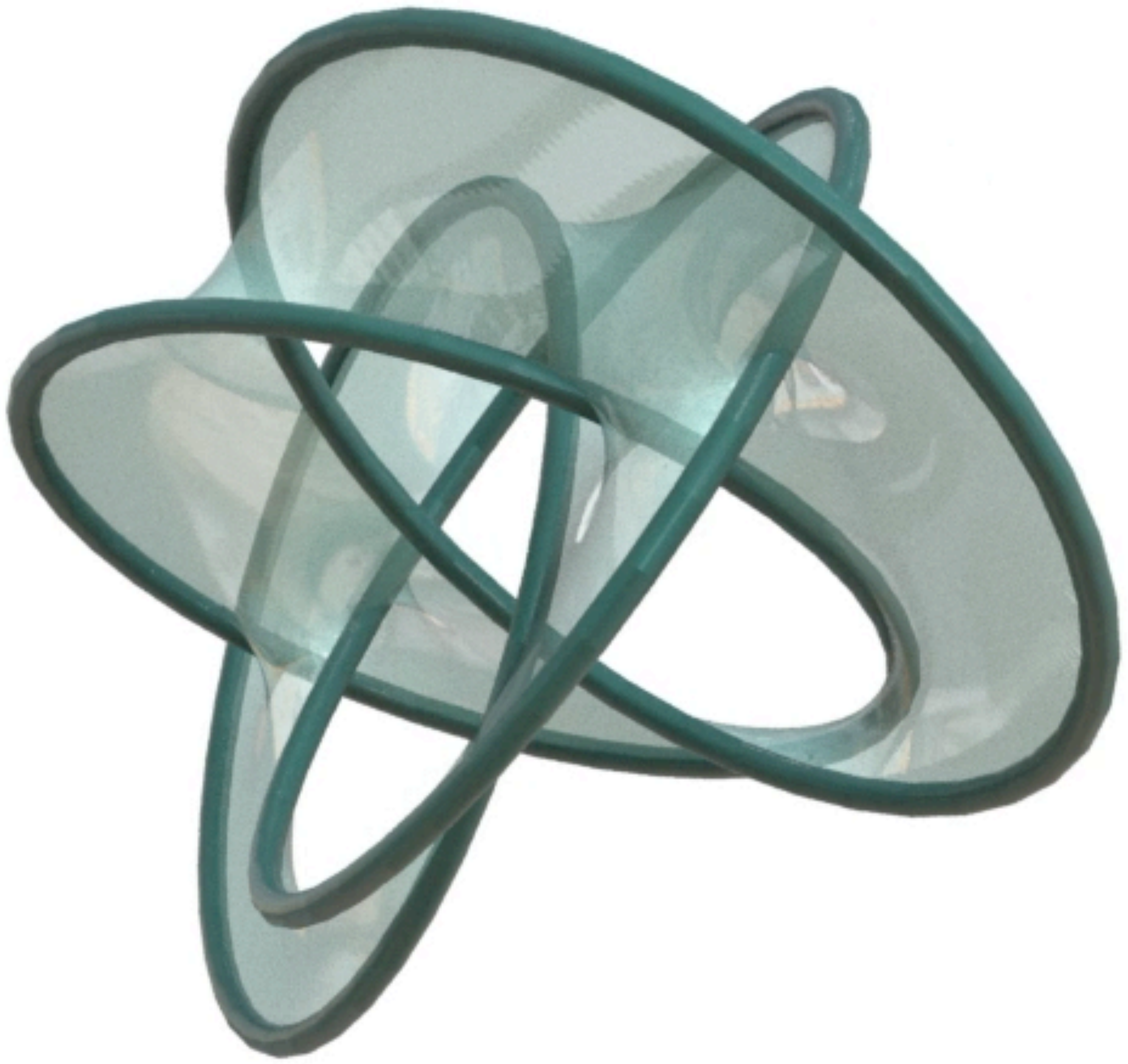

Thank you for your attention!

Stephanie Wang
stw006@eng.ucsd.edu

Project page: https://cseweb.ucsd.edu/~alchern/projects/MinimalCurrent/

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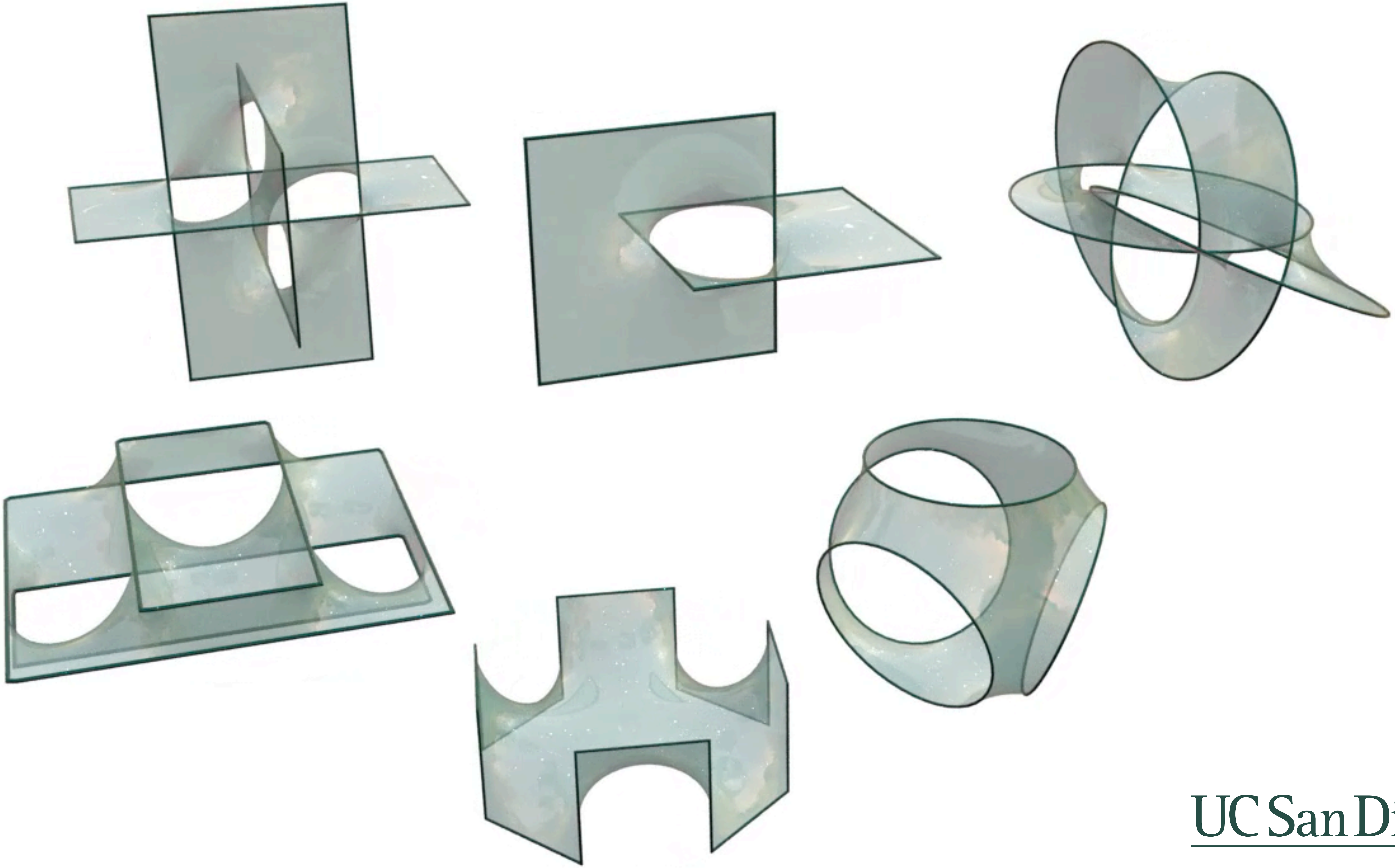

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